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### Correcting Estimation Bias in Dynamic Term Structure Models

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# Correcting Estimation Bias in Dynamic Term Structure Models

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The affine dynamic term structure model (DTSM) is the canonical empirical finance representation of the yield curve. However, the possibility that DTSM estimates may be distorted by small-sample bias has been largely ignored. We show that conventional estimates of DTSM coefficients are indeed severely biased, and this bias results in misleading estimates of expected future short-term interest rates and of long-maturity term premia. We provide a variety of bias-corrected estimates of affine DTSMs, for both maximally flexible and overidentified specifications. Our estimates imply interest rate expectations and term premia that are more plausible from a macrofinance perspective. This article has supplementary material online.

KEY WORDS: Small-sample bias correction; Term premium; Vector autoregression.

## 1. INTRODUCTION

The affine Gaussian dynamic term structure model (DTSM) is the canonical empirical finance representation of the yield curve, which is used to study a variety of questions about the interactions of asset prices, risk premia, and economic variables. One question of fundamental importance to both researchers and policy makers is to what extent movements in long-term interest rates reflect changes in expected future policy rates or changes in term premia. The answer to this question depends on the estimated dynamic system for the risk factors underlying yields, which, in affine DTSMs, is specified as a vector autoregression (VAR). Because of the high persistence of interest rates, maximum likelihood (ML) estimates of such models likely suffer from serious small-sample bias. Namely, interest rates will be spuriously estimated to be less persistent than they really are. While this problem has been recognized in the literature, no study, to date, has attempted to obtain bias-corrected estimates of a DTSM, quantify the extent of estimation bias, or assess the implications of that bias for economic inference. In this article, we provide a readily applicable methodology for bias-corrected estimation of both maximally flexible (exactly identified) and restricted (overidentified) affine DTSMs. Our estimates uncover significant bias in standard DTSM coefficient estimates and show that accounting for this bias substantially alters economic conclusions.

The bias in ML estimates of the VAR parameters in an affine DTSM parallels the well-known bias in ordinary least squares (OLS) estimates of autoregressive systems. Such estimates will generally be biased toward a dynamic system that displays less persistence than the true process. This bias is particularly severe when the estimation sample is small and the dynamic process is very persistent. Empirical DTSMs are invariably estimated under just such conditions, with data samples that contain only

a limited number of interest rate cycles. Hence, the degree of interest rate persistence is likely to be seriously underestimated. Consequently, expectations of future short-term interest rates will appear to revert too quickly to their unconditional mean, resulting in spuriously stable estimates of risk-neutral rates. Furthermore, the estimation bias that contaminates readings on expected future short-term rates also distorts estimates of long-maturity term premia.

While the qualitative implications of the small-sample DTSM estimation bias are quite intuitive, the magnitude of the bias and its impact on inference about expected short-rate paths and risk premia have been unclear. The ML methods, typically used to estimate DTSMs, were intensive, “hands-on” procedures because these models exhibited relatively flat likelihood surfaces with many local optima, as documented, among others, by Kim and Orphanides (2005), Hamilton and Wu (2012), and Christensen, Diebold, and Rudebusch (2011). The computational burden of estimation effectively precluded the application of simulation-based bias correction methods. However, recent work by Joslin, Singleton, and Zhu (2011) (henceforth JSZ) and Hamilton and Wu (2012) (henceforth HW) has shown that OLS can be used to solve a part of the estimation problem. We exploit these new procedures to facilitate bias-corrected estimation of DTSMs through repeated simulation and estimation. Specifically, we adapt the two-step estimation approaches of JSZ and HW by replacing the OLS estimates of the autoregressive system in the first step by simulation-based bias-corrected estimates. We then proceed with the second step of the estimation, which recovers the parameters determining the cross-sectional fit of the

model, in the usual way. This new estimation approach is a key methodological innovation of the article.

There are several different existing approaches to correct for small-sample bias in estimates of a VAR, including analytical bias approximations and bootstrap bias correction. While each of these could be applied to our present context, we use an indirect inference estimator for bias correction, which finds the data-generating process (DGP) parameters that lead to a mean of the OLS estimator equal to the original OLS estimates (Gourieroux, Renault, and Touzi 2000). This approach is closely related to the median-unbiased estimators of Andrews (1993) and Rudebusch (1992). One contribution of our article is to provide a new algorithm, based on results from the stochastic approximation literature, to quickly and reliably calculate the bias-correcting indirect inference estimator.

We first apply our methodology for bias-corrected DTSM estimation to the maximally flexible DTSM that was estimated in JSZ. In this setting, the ML estimates of the VAR parameters are exactly recovered by OLS. Using the authors' same model specifications and data samples, we quantify the bias in the reported parameter estimates and describe the differences in the empirical results when the parameters governing the factor dynamics are replaced with bias-corrected estimates. We find a very large estimation bias in JSZ. That is, the conventional estimates of JSZ imply a severe overestimation of the speed of interest rate mean reversion. As a result, the decomposition of long-term interest rates into expectations and risk premium components differs in statistically and economically significant ways between conventional ML and bias-corrected estimates. Risk-neutral forward rates, that is, short-term interest rate expectations, are substantially more volatile after correction for estimation bias, and they show a pronounced decrease over the last 20 years, consistent with lower longer-run inflation and interest rate expectations documented in the literature (Kozicki and Tinsley 2001; Kim and Orphanides 2005; Wright 2011). Furthermore, bias correction leads to term premia that are elevated around recessions and subdued during expansions, consistent with much theoretical and empirical research that supports countercyclical risk compensation.

In our second empirical application, we estimate a DTSM with overidentifying restrictions. HW showed that any affine DTSM—maximally flexible or overidentified—can be estimated by first obtaining reduced-form parameters using OLS and then calculating all structural parameters via minimum-chi-squared estimation. Many estimated DTSMs impose parameter restrictions to avoid overfitting and to facilitate numerical optimization of the likelihood function (examples include Ang and Piazzesi 2003; Kim and Wright 2005; Joslin, Priebsch, and Singleton 2010). Restrictions on risk pricing (Cochrane and Piazzesi 2008; Joslin, Priebsch, and Singleton 2010; Bauer 2011) have an additional benefit: they exploit the no-arbitrage condition, which ties the cross-sectional behavior of interest rates to their dynamic evolution, to help pin down the estimates of the parameters of the dynamic system. In this way, such restrictions could potentially reduce the bias in the estimates of these parameters. However, we find that the bias in ML estimates of a DTSM with risk price restrictions is large, indeed, similar in magnitude to our results for the maximally flexible model. Although this result may not generalize to all restricted models,

it shows that simply zeroing out some risk price parameters will not necessarily eliminate estimation bias.

There are a number of articles in the literature that are related to ours. Several studies attempt to indirectly reduce the bias in DTSM estimates—using risk price restrictions (see above), survey data (Kim and Orphanides 2005; Kim and Wright 2005), or near-cointegrated VAR specifications (Jardet, Monfort, and Pegoraro 2011)—but do not quantify the bias nor provide evidence as to how much it is reduced. Another group of articles has performed simulation studies to show the magnitude of the bias in DTSM estimates relative to some stipulated DGP (Ball and Torous 1996; Duffee and Stanton 2004; Kim and Orphanides 2005). These studies demonstrate that small-sample bias can be an issue using simulation studies, but do not quantify its magnitude or assess its implications for models estimated on real data. The most closely related is Phillips and Yu (2009), which also performs bias-corrected estimation of asset pricing models. This analysis parallels ours in that the authors also use simulation-based bias correction and show the economic implications of correcting for small-sample bias. However, their focus differed from ours in that they aimed at reducing the bias in prices of contingent claims, whereas we address a different economic question, namely the implications of small-sample bias on estimated policy expectations and nominal term premia.

Our article is structured as follows. Section 2 describes the model, the econometric problems with conventional estimation, and the intuition of our methodology for bias-corrected estimation. In Section 3, we discuss OLS estimates of interest rate VARs and the improvements from bias-corrected estimates. In Section 4, we describe how to estimate maximally flexible models with bias correction, apply this methodology to the model of JSZ, and discuss the statistical and economic implications. We also perform a simulation study to systematically assess the value of bias correction in such a context. In Section 5, we show how to perform bias-corrected estimation for restricted models, adapting the methodology of HW, and apply this approach to a model with restrictions on the risk pricing. Section 6 concludes.

## 2. ESTIMATION OF AFFINE MODELS

In this section, we set up a standard affine Gaussian DTSM, and describe the econometric issues, including small-sample bias, that arise due to the persistence of interest rates. Then, we discuss recent methodological advances and how they make bias correction feasible.

### 2.1 Model Specification

The discrete-time affine Gaussian DTSM, the workhorse model in the term structure literature since Ang and Piazzesi (2003), has three key elements. First, a vector of  $N$  risk factors,  $\mathbf{X}_t$ , follows a first-order Gaussian VAR under the objective probability measure  $\mathbb{P}$ :

$$\mathbf{X}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{X}_t + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}, \quad (1)$$

where  $\boldsymbol{\varepsilon}_t \sim N(0, \mathbf{I}_N)$  and  $\boldsymbol{\Sigma}$  is lower triangular. Time  $t$  is measured in months throughout the article. Second, the short-term

interest rate  $r_t$ , is an affine function of the pricing factors:

$$r_t = \delta_0 + \delta_1' \mathbf{X}_t. \quad (2)$$

Third, the stochastic discount factor (SDF) that prices all assets under the absence of arbitrage is of the essentially affine form (Duffee 2002):

$$-\log(M_{t+1}) = r_t + \frac{1}{2} \lambda_t' \lambda_t + \lambda_t' \varepsilon_{t+1},$$

where the  $N$ -dimensional vector of risk prices is affine in the pricing factors,

$$\lambda_t = \lambda_0 + \lambda_1 \mathbf{X}_t,$$

for  $N$ -vector  $\lambda_0$  and  $N \times N$  matrix  $\lambda_1$ . As a consequence of these assumptions, a risk-neutral probability measure  $\mathbb{Q}$  exists such that the price of an  $m$ -period default-free zero coupon bond is  $P_t^m = E_t^{\mathbb{Q}}(e^{-\sum_{h=0}^{m-1} r_{t+h}})$ , and under  $\mathbb{Q}$ , the risk factors also follow a Gaussian VAR,

$$\mathbf{X}_{t+1} = \boldsymbol{\mu}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}} \mathbf{X}_t + \boldsymbol{\Sigma} \varepsilon_{t+1}^{\mathbb{Q}}. \quad (3)$$

The prices of risk determine how the change of measure affects the VAR parameters:

$$\boldsymbol{\mu}^{\mathbb{Q}} = \boldsymbol{\mu} - \boldsymbol{\Sigma} \lambda_0, \quad \boldsymbol{\Phi}^{\mathbb{Q}} = \boldsymbol{\Phi} - \boldsymbol{\Sigma} \lambda_1. \quad (4)$$

Bond prices are exponentially affine functions of the pricing factors:

$$P_t^m = e^{\mathcal{A}_m + \mathcal{B}_m' \mathbf{X}_t},$$

with loadings  $\mathcal{A}_m = \mathcal{A}_m(\boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \delta_0, \delta_1, \boldsymbol{\Sigma})$  and  $\mathcal{B}_m = \mathcal{B}_m(\boldsymbol{\Phi}^{\mathbb{Q}}, \delta_1)$  that follow the recursions

$$\mathcal{A}_{m+1} = \mathcal{A}_m + (\boldsymbol{\mu}^{\mathbb{Q}})' \mathcal{B}_m + \frac{1}{2} \mathcal{B}_m' \boldsymbol{\Sigma} \boldsymbol{\Sigma}' \mathcal{B}_m - \delta_0,$$

$$\mathcal{B}_{m+1} = (\boldsymbol{\Phi}^{\mathbb{Q}})' \mathcal{B}_m - \delta_1,$$

with starting values  $\mathcal{A}_0 = 0$  and  $\mathcal{B}_0 = 0$ . Model-implied yields are  $y_t^m = -m^{-1} \log P_t^m = A_m + B_m' X_t$ , with  $A_m = -m^{-1} \mathcal{A}_m$  and  $B_m = -m^{-1} \mathcal{B}_m$ . Risk-neutral yields, the yields that would prevail if investors were risk-neutral, can be calculated using

$$\tilde{y}_t^m = \tilde{A}_m + \tilde{B}_m' \mathbf{X}_t, \quad \tilde{A}_m = -m^{-1} \mathcal{A}_m(\boldsymbol{\mu}, \boldsymbol{\Phi}, \delta_0, \delta_1, \boldsymbol{\Sigma}),$$

$$\tilde{B}_m = -m^{-1} \mathcal{B}_m(\boldsymbol{\Phi}, \delta_1).$$

Risk-neutral yields reflect policy expectations over the lifetime of the bond,  $m^{-1} \sum_{h=0}^{m-1} E_t r_{t+h}$ , plus a time-constant convexity term. The *yield term premium* is defined as the difference between actual and risk-neutral yields,  $yt p_t^m = y_t^m - \tilde{y}_t^m$ . Model-implied forward rates for loans starting at  $t+n$  and maturing at  $t+m$  are given by  $f_t^{n,m} = (m-n)^{-1} (\log P_t^m - \log P_t^n) = (m-n)^{-1} (m y_t^m - n y_t^n)$ . Risk-neutral forward rates  $\tilde{f}_t^{n,m}$  are calculated, in an analogous fashion, from risk-neutral yields. The *forward term premium* is defined as  $ft p_t^{n,m} = f_t^{n,m} - \tilde{f}_t^{n,m}$ .

The appeal of a DTSM is that all yields, forward rates, and risk premia are functions of a small number of risk factors. Let  $M$  be the number of yields in the data used for estimation. The  $M$ -vector of model-implied yields is  $\mathbf{Y}_t = \mathbf{A} + \mathbf{B} \mathbf{X}_t$ , with  $\mathbf{A} = (A_{m_1}, \dots, A_{m_M})'$  and  $\mathbf{B} = (\mathcal{B}_{m_1}, \dots, \mathcal{B}_{m_M})'$ . A low-dimensional model will not have a perfect empirical fit for all yields, so we specify observed yields to include a measurement error,  $\hat{\mathbf{Y}}_t = \mathbf{Y}_t + \mathbf{e}_t$ . While measurement error can potentially have

serial correlation (Adrian, Crump, and Moench 2012; Hamilton and Wu *in press*), we follow much of the literature and take  $\mathbf{e}_t$  to be an iid process.

As in JSZ and HW, we assume that  $N$  linear combinations of yields are priced without error. Specifically, we take the first three principal components of yields as risk factors. Denote by  $\mathbf{W}$  the  $3 \times M$  matrix that contains the eigenvectors corresponding to the three largest eigenvalues of the covariance matrix of  $\hat{\mathbf{Y}}_t$ . By assumption,  $\mathbf{X}_t = \mathbf{W} \mathbf{Y}_t = \mathbf{W} \hat{\mathbf{Y}}_t$ . Generally, risk factors can be unobserved factors (which are filtered from observed variables), observables such as yields or macroeconomic variables, or any combination of unobserved and observable factors. Our estimation method is applicable to cases with observable and/or unobservable yield curve factors, as well as to macro-finance DTSMs (Ang and Piazzesi 2003; Rudebusch and Wu 2008; Joslin, Priebsch, and Singleton 2010). The only assumption that is necessary for our method to be applicable is that  $N$  linear combinations of risk factors are priced without error.

One possible parameterization of the model is in terms of  $\boldsymbol{\gamma} = (\boldsymbol{\mu}, \boldsymbol{\Phi}, \boldsymbol{\mu}^{\mathbb{Q}}, \boldsymbol{\Phi}^{\mathbb{Q}}, \delta_0, \delta_1, \boldsymbol{\Sigma})$ , leaving aside the parameters determining the measurement error distribution. Given  $\boldsymbol{\gamma}$ , the risk sensitivity parameters  $\lambda_0$  and  $\lambda_1$  follow from Equation (4). Model identification requires normalizing restrictions (Dai and Singleton 2000). For example,  $\boldsymbol{\gamma}$  has 34 free elements in a three-factor model, but only 22 parameters are identified, so at least 12 normalizing restrictions are necessary. If the model is exactly identified, one speaks of a "maximally flexible" model, as opposed to an overidentified model, in which additional restrictions are imposed.

## 2.2 Maximum Likelihood Estimation and Small-Sample Bias

While it is conceptually straightforward to calculate the ML estimator (MLE) of  $\boldsymbol{\gamma}$ , this has been found to be very difficult in practice. The list of studies that have documented such problems is long and includes Ang and Piazzesi (2003), Kim and Orphanides (2005), Duffee (2011a), Christensen, Diebold, and Rudebusch (2009, 2011), and Hamilton and Wu (2012). The first issue is to numerically find the MLE, which is problematic since the likelihood function is high dimensional, badly behaved, and typically exhibits local optima (with different economic implications). The second issue is the considerable statistical uncertainty around the point estimates of DTSM parameters (Kim and Orphanides 2005; Rudebusch 2007; Bauer 2011). The third issue, which is the focus of this article, is that the MLE suffers from small-sample bias (Ball and Torous 1996; Duffee and Stanton 2004; Kim and Orphanides 2005). All three of these problems are related to the high persistence of interest rates, which complicates the inference about the VAR parameters. Intuitively, because interest rates revert to their unconditional mean very slowly, a data sample will typically contain only very few interest rate cycles, which makes it difficult to infer  $\boldsymbol{\mu}$  and  $\boldsymbol{\Phi}$ . The likelihood surface is rather flat in certain dimensions around its maximum; thus, numerical optimization is difficult and statistical uncertainty is high. Furthermore, the severity of the small-sample bias depends positively on the persistence of process.



The economic implications of small-sample bias are likely to be important, because the VAR parameters determine the risk-neutral rates and term premia. Since the bias causes the speed of mean reversion to be overestimated, model-implied interest rate forecasts will tend to be too close to their unconditional mean, especially at long horizons. Therefore, risk-neutral rates will be too stable, and too large a portion of the movements in nominal interest rates will be attributed to movements in term premia.

### 2.3 Bias Correction for DTSMs

Simulation-based bias correction methods require repeated sampling of new datasets and calculation of the estimator. For this to be computationally feasible, the estimator needs to be calculated quickly and reliably for each simulated dataset. In the DTSM context, MLE has typically involved high computational cost, with low reliability in terms of finding a global optimum, which effectively precluded simulation-based bias correction.

However, two important recent advances in the DTSM literature substantially simplify the estimation of affine Gaussian DTSMs. First, JSZ proved that in a maximally flexible model, the MLE of  $\mu$  and  $\Phi$  can be obtained using OLS. Second, HW showed that any affine Gaussian model can be estimated by first estimating a reduced form of the model by OLS and then finding the structural parameters by minimizing a chi-squared statistic. Given these methodological innovations, there is no need to maximize a high-dimensional, badly behaved likelihood function. Estimation can be performed by a consistent and efficient two-stage estimation procedure, where the first stage consists of OLS and the second stage involves finding the remaining (JSZ) or the structural (HW) parameters, without minimal computational difficulties.

These methodological innovations make correction for small-sample bias feasible, because of their use of linear regressions to solve a part of the estimation problem. In both approaches, a VAR system is estimated in the first stage, which is the place where bias correction is needed. We propose to apply bias correction techniques to the estimation of the VAR parameters, and to carry out the rest of the estimation procedure in the normal fashion. This, in a nutshell, is the methodology that we will use in this article.

Before detailing our approach in Sections 4 (for maximally flexible models) and 5 (for overidentified models), we first discuss how to obtain bias-corrected estimates of VAR parameters, as well as the particular features of the VARs in term structure models.

## 3. BIAS CORRECTION FOR INTEREST RATE VARs

This section describes interest rate VARs, that is, VARs that include interest rates or factors derived from interest rates, and discusses small-sample OLS bias and bias correction in this context.

### 3.1 Characteristic Features of Interest Rate VARs

The factors in a DTSM include either individual interest rates, linear combinations of these, or latent factors that are filtered from the yield curve, and hence will typically have similar

statistical properties as individual interest rates. The amount of persistence displayed by both nominal and real interest rates is extraordinarily high. First-order autocorrelation coefficients are typically close to one, and unit root tests often do not reject the null of a stochastic trend. However, economic arguments strongly suggest that interest rates are stationary. A unit root is implausible, since nominal interest rates generally do not turn negative and remain within some limited range, and an explosive root (exceeding one) is unreasonable since forecasts would diverge. For these reasons, empirical DTSMs almost invariably assume stationarity by implicitly or explicitly imposing the constraint that all roots of the factor VAR are less than one in absolute value. In a frequentist setting, the parameter space is restricted appropriately, while in a Bayesian framework (Ang, Boivin, and Dong 2009; Bauer 2011), stationarity is incorporated in the prior. In this article, we assume that interest rates are stationary.

The data samples used in the estimation of interest rate VARs are typically rather small. Researchers often start their samples in the 1980s or later because of data availability or potential structural breaks (e.g., Joslin, Priebisch, and Singleton 2010; Joslin, Singleton, and Zhu 2011; Wright 2011). Even if one goes back to the 1960s (e.g., Cochrane and Piazzesi 2005; Duffee 2011b), the sample can be considered rather small in light of the high persistence of interest rates—there are only few interest rate cycles, and uncertainty around the VAR parameters remains high. Notably, it does not matter whether one samples at quarterly, monthly, weekly, or daily frequency: sampling at a higher frequency increases not only the sample length but also the persistence (Pierse and Snell 1995).

Researchers attempting to estimate DTSMs are thus invariably faced with highly persistent risk factors and small available data samples to infer the dynamic properties of the model.

### 3.2 Small-Sample Bias of OLS

Consider the VAR system in Equation (1). We focus our exposition on a first-order VAR since the extension to higher-order models is straightforward. We assume that the VAR is stationary, that is, all the eigenvalues of  $\Phi$  are less than one in modulus. The parameters of interest are  $\theta = \text{vec}(\Phi)$ . Denote the true values by  $\theta_0$ . The MLE of  $\theta$  can be obtained by applying OLS to each equation of the system (Hamilton 1994, chap. 11.1). Let  $\hat{\theta}_T$  denote the OLS estimator, and  $\hat{\theta}$  the estimates from a particular sample.

Because of the presence of lagged endogenous variables, the assumption of strict exogeneity is violated, and the OLS estimator is biased in finite samples, that is,  $E(\hat{\theta}_T) \neq \theta_0$ . The bias function  $b_T(\theta) = E(\hat{\theta}_T) - \theta$  relates the bias of the OLS estimator to the value of the data-generating  $\theta$ . Because  $\hat{\theta}_T$  is distributionally invariant with respect to  $\mu$  and  $\Sigma$ , the bias function depends only on  $\theta$  and not on  $\mu$  or  $\Sigma$ —the proof of distributional invariance for the univariate case in Andrews (1993) naturally extends to VAR models. The bias in  $\hat{\theta}_T$  is more severe, the smaller the available sample and the more persistent the process is (e.g., see Nicholls and Pope 1988). Hence, for interest rate VARs, the bias is potentially sizeable. The consequence is that OLS tends to underestimate the persistence of the system, as measured, for example, by the largest eigenvalue of

$\Phi$  or by the half-life of shocks. In that case, forecasts revert to the unconditional mean too quickly.

In principle, one can alternatively define bias using the median as the relevant central tendency of an estimator. Some authors, including Andrews (1993) and Rudebusch (1992), have argued that median-unbiased estimators have useful impartiality properties, given that the distribution of the OLS estimator can be highly skewed in autoregressive models for persistent processes. However, for a vector-valued random variable, the median is not uniquely defined, because orderings of multivariate observations are not unique. One possible definition is to use the element-by-element median as in Rudebusch (1992), and the working paper version of this article (Bauer, Rudebusch, and Wu 2012) includes estimation results based on correcting for median bias defined in this way—they are very similar to the results presented here. We focus on mean bias, since its statistical foundation is more sound in the context of inference about a vector-valued parameter.

### 3.3 Methods for Bias Correction

The aim of all bias correction methods is to estimate the value of the bias function, that is,  $b_T(\theta_0)$ . We now discuss alternative analytical and simulation-based approaches for this purpose.

**3.3.1 Analytical Bias Approximation.** The statistical literature has developed analytical approximations for the mean bias in univariate autoregressions (Kendall 1954; Marriott and Pope 1954; Stine and Shaman 1989) and in VARs (Nicholls and Pope 1988; Pope 1990), based on approximations of the small-sample distribution of the OLS estimator. These closed-form solutions are fast and easy to calculate, and are accurate up to first order. They have been used for obtaining more reliable VAR impulse responses by Kilian (1998a, 2011), and in finance applications by Amihud, Hurvich, and Wang (2009) and Engsted and Pedersen (2012), among others.

**3.3.2 Bootstrap Bias Correction.** Simulation-based bias correction methods rely on the bootstrap to estimate the bias. Data are simulated using a (distribution-free) residual bootstrap, taking the OLS estimates as the data-generating parameters, and the OLS estimator is calculated for each simulated data sample. Comparing the mean of these estimates with  $\hat{\theta}$  provides an estimate of  $b_T(\hat{\theta})$ , which approximates the bias at the true data-generating parameters,  $b_T(\theta_0)$ . Hence, bootstrap bias correction removes first-order bias as does analytical bias correction, and both methods are asymptotically equivalent. Applications in time series econometrics include Kilian (1998b, 1999). Prominent examples of the numerous applications in finance are Phillips and Yu (2009) and Tang and Chen (2009). For a detailed description of bootstrap bias correction, see the online Appendix A.

**3.3.3 Indirect Inference.** Analytical and bootstrap bias correction estimate the bias function at  $\hat{\theta}$ , whereas the true bias is equal to the value of the bias function at  $\theta_0$ . The fact that these generally differ motivates a more refined bias correction procedure. For removing higher-order bias and improving accuracy further, one possibility is to iterate on the bootstrap bias correction, as suggested by Hall (1992). However, the

computational burden of this “iterated bootstrap” quickly becomes prohibitively costly.

An alternative is to choose the value of  $\theta$  by inverting the mapping from DGP parameters to the mean of the OLS estimator. In other words, one picks that  $\theta$  which, if taken as the data-generating parameter vector, leads to a mean of the OLS estimator equal to  $\hat{\theta}$ . This bias correction method removes first- and higher-order bias. The resulting estimator is a special case of the indirect inference estimator of Gourieroux, Monfort, and Renault (1993), as detailed in Gourieroux, Renault, and Touzi (2000). It is also closely related to the median-unbiased estimators of Andrews (1993) and Rudebusch (1992).

Calculation of this estimator requires the inversion of the unknown mapping from DGP parameters to the mean of the OLS estimator. The residual bootstrap provides measurements of this mapping for given values of  $\theta$ . Since the use of the bootstrap introduces a stochastic element, one cannot use conventional numerical root-finding methods. However, the stochastic approximation literature has developed algorithms to find the root of functions that are measured with error. We adapt an existing algorithm to our present context, which allows us to efficiently and reliably calculate our bias-corrected estimators. The idea behind the algorithm is the following: For each iteration, simulate a small set of bootstrap samples using some “trial” DGP parameters, and calculate the mean of the OLS estimator and the distance to the target (the OLS estimates in the original data). In the following iteration, adjust the DGP parameters based on this distance. After a fixed number of iterations, take the average of the DGP parameters over all iterations, discarding some initial values. This average has desirable convergence properties and will be close to the true solution. For the formal definition of our bias-corrected estimator and for a detailed description of the algorithm and underlying assumptions, refer to the online Appendix B. While indirect inference estimators are well understood theoretically and have been applied successfully in practice, they are often difficult to calculate. Our algorithmic implementation is generally applicable and can be used to calculate other indirect inference estimators.

In the online Appendix C, we compare alternative bias correction methods for VAR estimation with OLS and with each other, and show that our indirect inference approach performs well.

### 3.4 Eigenvalue Restrictions

We assume stationarity of the VAR; hence, we need to ensure that estimates of  $\Phi$  have eigenvalues that are less than one in modulus, that is, the VAR has only stationary roots. In the context of bias correction, this restriction is particularly important, because bias-corrected VAR estimates exhibit explosive roots much more frequently than OLS estimates (as is evident in our simulation studies). In a DTSM, one might impose the tighter restriction that the largest eigenvalue of  $\Phi$  does not exceed the largest eigenvalue of  $\Phi^Q$ , to ensure that policy rate forecasts are no more volatile than forward rates. Either way, it will generally be necessary to impose restrictions on the set of possible eigenvalues.

In this article, we impose the restriction that bias-corrected estimates are stationary using the stationarity adjustment

suggested in Kilian (1998b). If the bias-corrected estimates have explosive roots, the bias estimate is shrunk toward zero until the restriction is satisfied. This procedure is simple, fast, and effective. It is also flexible: We can impose any restriction on the largest eigenvalue of  $\Phi$ , as long as the OLS estimates satisfy this restriction.

#### 4. ESTIMATION OF MAXIMALLY FLEXIBLE MODELS

In this section, we describe our methodology to obtain bias-corrected estimates of maximally flexible models. We apply this approach to the empirical setting of JSZ, quantify the small-sample bias in their model estimates, and assess the economic implications of bias correction. Although the approach that we develop in Section 5 is more general and could be applied here, adapting the estimation framework of JSZ has two advantages. First, we start from a well-understood benchmark, namely the ML estimates of the affine model parameters. Second, our numerical results are directly comparable with those of JSZ.

##### 4.1 Estimation Methodology

We assume that there are no overidentifying restrictions and, as mentioned above, that  $N$  linear combinations of yields are exactly priced by the model. Under these assumptions, any affine Gaussian DTSM is equivalent to one where the pricing factors  $\mathbf{X}_t$  are taken to be those linear combinations of yields. The MLE can be obtained by first estimating the VAR parameters  $\mu$  and  $\Phi$  using OLS, and then maximizing the likelihood function for given values of  $\mu$  and  $\Phi$  (as shown by JSZ). This suggests a natural way to obtain bias-corrected estimates of the DTSM parameters: first obtain bias-corrected estimates of the VAR parameters, and then proceed with the estimation of the remaining parameters as usual. This, in a nutshell, is the approach we propose here.

The normalization suggested by JSZ parameterizes the model in terms of  $(\mu, \Phi, \Sigma, r_{\infty}^{\mathbb{Q}}, \lambda^{\mathbb{Q}})$ , where  $r_{\infty}^{\mathbb{Q}}$  is the risk-neutral unconditional mean of the short-term interest and the  $N$ -vector  $\lambda^{\mathbb{Q}}$  contains the eigenvalues of  $\Phi^{\mathbb{Q}}$ . What characterizes this normalization is that (1) the model is parameterized in terms of physical dynamics and risk-neutral dynamics, and (2) all the normalizing restrictions are imposed on the risk-neutral dynamics. It is particularly useful because of the separation result that follows: the joint likelihood function of observed yields can be written as the product of (1) the “ $\mathbb{P}$ -likelihood,” the conditional likelihood of  $\mathbf{X}_t$ , which depends only on  $(\mu, \Phi, \Sigma)$ , and (2) the “ $\mathbb{Q}$ -likelihood,” the conditional likelihood of the yields, which depends only on  $(r_{\infty}^{\mathbb{Q}}, \lambda^{\mathbb{Q}}, \Sigma)$  and the parameters for the measurement errors. Because of this separation, the values of  $(\mu, \Phi)$  that maximize the joint likelihood function are the same as the ones that maximize the  $\mathbb{P}$ -likelihood, namely the OLS estimates. This gives rise to the simple two-step estimation procedure suggested by JSZ.

The OLS estimates of the VAR parameters, denoted by  $(\hat{\mu}, \hat{\Phi})$ , suffer from the small-sample bias that plagues all least-square estimates of autoregressive systems. To deal with this problem, we obtain bias-corrected estimates, denoted by  $(\tilde{\mu}, \tilde{\Phi})$ . Here, we focus on the indirect inference estimator described above (using 6000 iterations, of which we discard the first 1000,

with 50 bootstrap samples in each iteration and an adjustment parameter  $\alpha_i = 0.5$ ). We present results for analytical and bootstrap bias correction in the online Appendix D. Because of the JSZ separation result, our first-step estimates are independent of the parameter values that, in the second step, maximize the joint likelihood function. Differently put, we do not have to worry in the first step about cross-sectional fit. We estimate the remaining parameters by maximizing the joint likelihood function over  $(r_{\infty}^{\mathbb{Q}}, \lambda^{\mathbb{Q}}, \Sigma)$ , fixing the values of  $\mu$  and  $\Phi$  at  $\tilde{\mu}$  and  $\tilde{\Phi}$ . This procedure will take care of the small-sample estimation bias, while achieving similar cross-sectional fit as MLE.

To calculate standard errors for  $\tilde{\mu}$  and  $\tilde{\Phi}$ , we use the conventional asymptotic approximation, and simply plug the bias-corrected point estimates into the usual formula for OLS standard errors. Alternative approaches using bootstrap simulation are possible, but for the present context, we deem this pragmatic solution sufficient. For the estimates of  $(r_{\infty}^{\mathbb{Q}}, \lambda^{\mathbb{Q}}, \Sigma)$ , we calculate quasi-MLE standard errors, approximating the gradient and Hessian of the likelihood function numerically.

##### 4.2 Data and Parameter Estimates

We first replicate the estimates of JSZ and then assess the implications of bias correction. We focus on their “RPC” specification, in which the pricing factors  $\mathbf{X}_t$  are the first three principal components of yields and  $\Phi^{\mathbb{Q}}$  has distinct real eigenvalues. There are no overidentifying restrictions; thus, there are 22 free parameters, not counting measurement error variances. The free parameters are  $\mu$  (3),  $\Phi$  (9),  $r_{\infty}^{\mathbb{Q}}$  (1),  $\lambda^{\mathbb{Q}}$  (3), and  $\Sigma$  (6). The monthly dataset of zero-coupon Treasury yields from January 1990 to December 2007, with yield maturities of 6 months and 1, 2, 3, 5, 7, and 10 years, is available on Ken Singleton’s website.

To obtain the MLE, we follow the estimation procedure of JSZ, and we denote this set of estimates by “OLS.” Then, we apply bias correction, denoting the resulting estimates by “BC.” Table 1 shows point estimates and standard errors for the DTSM parameters. The OLS estimates in the left panel exactly correspond to the ones reported in JSZ. The bias-corrected estimates are reported in the right panel. Because of the JSZ separation result, the estimated risk-neutral dynamics and the estimated  $\Sigma$  are very similar across the two sets of estimates—slight differences stem from the fact that  $\Sigma$  enters both the  $\mathbb{P}$ -likelihood and the  $\mathbb{Q}$ -likelihood, wherefore different values of  $\mu$  and  $\Phi$  lead to different optimal values of  $(r_{\infty}^{\mathbb{Q}}, \lambda^{\mathbb{Q}}, \Sigma)$  in the second stage. The cross-sectional fit is also basically identical, with a root mean squared fitting error of about six basis points. The estimated VAR dynamics are however substantially different with and without bias correction.

##### 4.3 Economic Implications of Bias Correction

To assess the economic implications of bias-corrected DTSM estimates, we first consider measures of persistence of the estimated VAR, shown in the top panel of Table 2. The first row reports the maximum absolute eigenvalue of the estimated  $\Phi$ , which increases significantly when bias correction is applied. The statistics in the second and third rows are based on the impulse response function (IRF) of the level factor (the first principal component) to a level shock. The second row shows

Table 1. Maximally flexible DTSM—parameter estimates

	OLS			BC		
1200 $\mu$	-0.5440 (0.2330)	-0.1263 (0.0925)	0.0700 (0.0398)	-0.3290 (0.2358)	-0.2321 (0.0931)	0.0738 (0.0401)
$\Phi$	0.9788 (0.0111)	0.0133 (0.0392)	0.4362 (0.2127)	0.9987 (0.0112)	0.0006 (0.0397)	0.4721 (0.2152)
	0.0027 (0.0044)	0.9737 (0.0156)	0.3532 (0.0844)	-0.0020 (0.0044)	0.9926 (0.0019)	0.3462 (0.0366)
	-0.0025 (0.0019)	-0.0023 (0.0067)	0.8537 (0.0364)	0.0001 (0.0019)	0.0021 (0.0068)	0.8813 (0.0366)
$ \text{eig}(\Phi) $	0.9678	0.9678	0.8706	0.9991	0.9991	0.8744
1200 $r_{\infty}^Q$	8.6055 (0.6590)			8.6568 (0.6537)		
$\lambda^Q$	0.9976 (0.0004)	0.9519 (0.0082)	0.9287 (0.0145)	0.9976 (0.0004)	0.9518 (0.0082)	0.0287 (0.0145)
1200 $\Sigma$	0.6365 (0.0324)	0	0	0.6442 (0.0317)	0	0
	-0.1453 (0.0216)	0.2097 (0.0143)	0	-0.1468 (0.0216)	0.2107 (0.0146)	0
	0.0630 (0.0075)	-0.0117 (0.0072)	0.0867 (0.0048)	0.0635 (0.0076)	-0.0113 (0.0076)	0.0873 (0.0045)

NOTE: Parameter estimates for the DTSM as in JSZ. Left panel shows OLS/ML estimates, and right panel shows bias-corrected (BC) estimates.

the half-life, that is, the horizon at which the IRF falls below 0.5, calculated as in Kilian and Zha (2002). The half-life is 2 years for OLS, and about 22 years for BC. The third row reports the value of the IRF at the 5-year horizon, which is increased through bias correction by a factor of about 5–6. The results here show that OLS greatly understates the persistence of the dynamic system, since bias correction substantially increases the estimated persistence.

We now turn to risk-neutral rates and nominal term premia, focusing on a decomposition of the 1-month forward rate for a loan maturing in 4 years, that is,  $f_t^{47,48}$ . The last three rows of Table 2 show standard deviations of the model-implied forward rate and of its risk-neutral and term premium components. The volatility of the forward rate itself is the same across estimates, since the model fit is similar. The volatility of the risk-neutral forward rate is higher for the bias-corrected estimates than for OLS by a factor of about 3–4. The slower mean reversion leads to much more volatile risk-neutral rates. The mean BC estimates lead to a particularly high volatility of risk-neutral rates. The volatility of the forward term premium is similar across

estimates, with slightly more variability after bias correction. Figure 1 shows the alternative estimates of the risk-neutral forward rate in the top panel and the estimated forward term premia in the bottom panel. The differences are rather striking. The risk-neutral forward rate resulting from OLS estimates displays little variation, and the associated term premium closely mirrors the movements of the forward rate. The secular decline in the forward rate is attributed to the term premium, which does not show any discernible cyclical pattern. In contrast, the risk-neutral forward rates implied by bias-corrected estimates vary much more over time and account for a considerable portion of the secular decline in the forward rate. There is a pronounced cyclical pattern for both the risk-neutral rate and the term premium.

From a macrofinance perspective, the decomposition implied by bias-corrected estimates seems more plausible. The secular decline in risk-neutral rates is consistent with results from survey-based interest rate forecasts (Kim and Orphanides 2005) and far-ahead inflation expectations (Kozicki and Tinsley 2001; Wright 2011), which have drifted downward over the last 20 years. The bias-corrected term premium estimates display a pronounced countercyclical pattern, rising notably during recessions. Most macroeconomists believe that risk premia vary significantly at the business cycle frequency and behave in such a countercyclical fashion, given theoretical work such as Campbell and Cochrane (1999) and Wachter (2006) as well as empirical evidence from Harvey (1989) to Lustig, Roussanov, and Verdelhan (2010). In contrast, the OLS-estimated term premium is very stable and, if anything, appears to decline a bit during economic recessions.

This empirical application shows that the small-sample bias in a typical maximally flexible estimated DTSM, as the one in JSZ, is sizable and economically significant. Taking account of this bias leads to risk-neutral rates and term premia that are significantly different from the ones implied by MLE. Specifically, bias-corrected policy expectations show higher and

Table 2. Maximally flexible DTSM—summary statistics

	OLS	BC
$\max(\text{eig}(\Phi))$	0.9678	0.9991
Half-life	24.0	265.0
IRF at 5 years	0.16	0.93
$\sigma(f_t^{47,48})$	1.392	1.392
$\sigma(\tilde{f}_t^{47,48})$	0.388	1.635
$\sigma(ftp_t^{47,48})$	1.301	1.656

NOTE: Summary statistics for OLS and bias-corrected (BC) estimates of the DTSM as in JSZ. First row: maximum eigenvalue of the estimated  $\Phi$ . Second and third rows: half-life and value of the IRF at the 5-year horizon for the response of the level factor to a level shock. Last three rows show sample standard deviations of the fitted 47- to 48-month forward rates and of the corresponding risk-neutral forward rates and forward term premia.



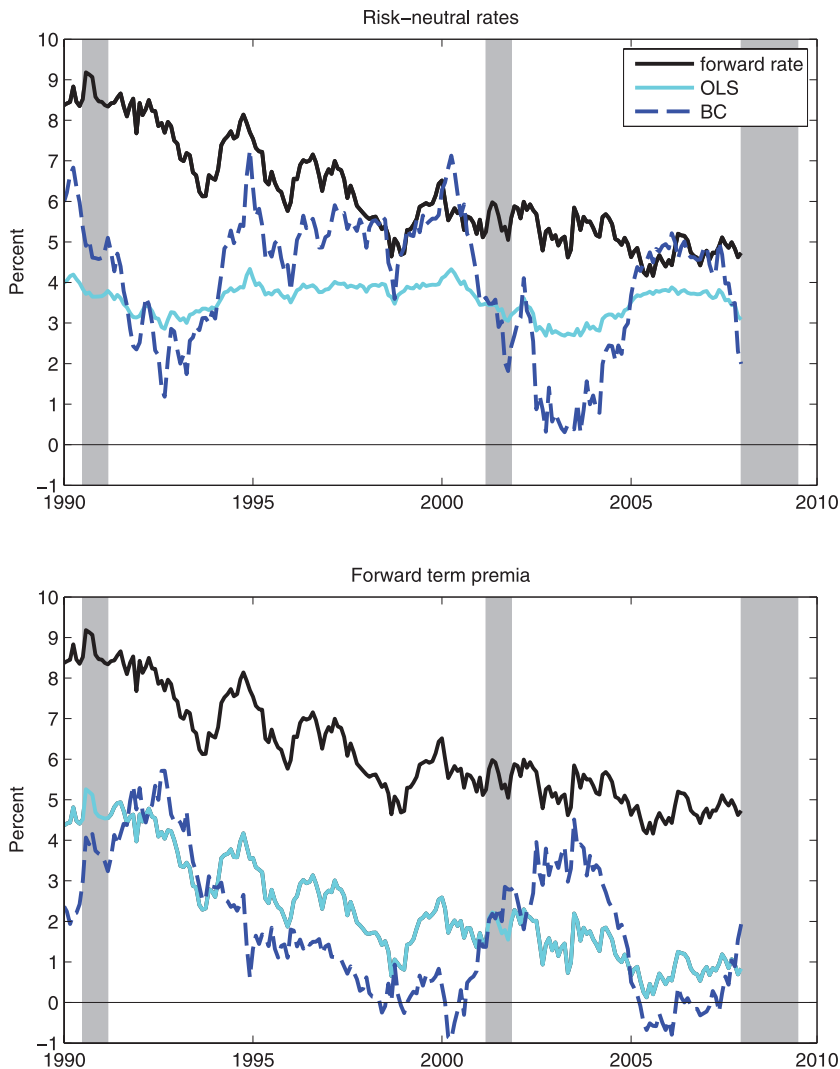


Figure 1. Maximally flexible DTSM—decomposition of forward rates. One-month forward rates with 4 years maturity, decomposed into risk-neutral forward rates and forward term premia using the affine Gaussian DTSM of JSZ. Sample: monthly observations from January 1990 to December 2007. Gray-shaded areas correspond to National Bureau of Economic Research (NBER) recessions. The online version of this figure is in color.

more plausible variation and contribute, to some extent, to the secular decline in long-term interest rates. Bias-corrected term premium estimates show a very pronounced countercyclical pattern, whereas conventional term premium estimates just parallel movements in long-term interest rates.

#### 4.4 Monte Carlo Study

Bias correction is designed to provide more accurate estimates of the model parameters, but the main objects of interest, risk-neutral rates and term premia, are highly nonlinear functions of these parameters. We use a Monte Carlo study to investigate whether bias-corrected estimation of a maximally flexible DTSM improves inference about the persistence of interest rates and about expected short-term interest rates and term premia. In addition, we evaluate how well out-of-sample forecasts based on bias-corrected estimates perform in comparison with alternative forecast methods.

We simulate 1000 yield datasets using the model specification described above and the bias-corrected parameter estimates

as the DGP. First, we simulate time series for  $\mathbf{X}_t$ , with  $T + H = 216 + 60 = 276$  observations from the VAR, drawing the starting values from their stationary distribution. Here,  $T$  is the sample used for estimation and  $H$  is the longest forecast horizon. Then, model-implied yields are calculated using the yield loadings for given DGP parameters  $(r_\infty^Q, \lambda^Q, \Sigma)$ , and taking  $\mathbf{W}$  as corresponding to the principal components in the original data. We add independent Gaussian measurement errors with a standard deviation of six basis points.

For each simulated dataset, we perform the same estimation procedures as above, obtaining OLS and BC estimates using the first  $T$  observations. Here, we run our estimation algorithm for 1500 iterations, discarding the first 500, using five bootstrap replications in each iteration and an adjustment parameter  $\alpha_i = 0.1$ . For bias-corrected estimates that have explosive roots, we apply the stationarity adjustment. In those cases where even the OLS estimates are explosive, we shrink the estimated  $\Phi$  matrix toward zero until it is stationary, and only then proceed to obtain bias-corrected estimates. Because of the very high persistence of the DGP process, bias-corrected estimates often imply explosive

Table 3. DTSM Monte Carlo study—summary statistics

	Mean			Median		
	DGP	OLS	BC	DGP	OLS	BC
$\max(\text{eig}(\Phi))$	0.9991	0.9875	0.9981	0.9991	0.9906	0.9996
Half-life	265.0	54.4	111.2	265.0	25.0	39.0
IRF at 5 years	0.93	0.27	0.53	0.93	0.21	0.55
$\sigma(f_i^{47,48})$	2.14	2.15	2.15	1.84	1.85	1.85
$\sigma(\tilde{f}_i^{47,48})$	2.76	1.87	2.60	2.47	1.49	2.36
$\sigma(ftp_i^{47,48})$	3.00	2.60	2.99	2.65	2.24	2.70
$\text{RMSE}(f_i^{47,48})$		0.02	0.02		0.01	0.01
$\text{RMSE}(\tilde{f}_i^{47,48})$		1.79	1.69		1.67	1.56
$\text{RMSE}(ftp_i^{47,48})$		1.79	1.70		1.70	1.56

NOTE: Summary statistics for persistence, variability, and accuracy of estimated rates and premia in DTSM Monte Carlo study. First three rows show true values (DGP) and means/medians of estimated values for the largest root of  $\Phi$ , the half-life in months (across estimates that have a half-life of less than 40 years), and the value of the IRF at the 5-year horizon for response of the first risk factor to own shocks. Rows four to six show means/medians of sample standard deviations of forward rates, risk-neutral forward rates, and forward premia. Last three rows show means/medians of RMSEs for estimated rates and premia. Volatilities and RMSEs are in annualized percentage points. For details refer to text.

VAR dynamics—the frequency of explosive eigenvalues before the stationarity adjustment is 74.7%. The OLS estimates have an explosive root in 13.9% of the replications.

The model parameters governing the VAR system are estimated with a substantial bias when using OLS, while BC estimates display a much smaller bias, as expected. The remaining parameters of the DTSM are estimated with similar accuracy in either case. As expected, Q-measure parameters are pinned down with high precision by the data, while inference about P-measure parameters is troublesome. The estimates of the model parameters are presented and discussed further in the online Appendix E.

Table 3 summarizes how accurate alternative estimates recover the main objects of interest. The first three rows show measures of persistence for the true parameters (DGP) and means and medians of these measures for the estimated parameters. As before, we calculate the largest absolute eigenvalue of  $\Phi$ , the half-life, and the value of the IRF at the 5-year horizon for the response of the level factor to own shocks. For the half-life, means/medians are calculated only across those replications for which the estimates imply a half-life of less than 40 years, the cutoff for our half-life calculation (Kilian and Zha 2002).

As expected, the persistence of the VAR is significantly underestimated by OLS, with central tendencies of the estimated persistence measures significantly below their true value. Bias-corrected estimation leads to much better results: it does not perfectly recover the true persistence, but the estimated persistence is higher than that for OLS and closer to the true model.

How accurately do the estimates capture policy expectations and term premia? We decompose the 4-year forward rate into expectations and risk premium components, for the true DGP parameters and for each set of estimated parameters. Rows four to six of Table 3 show the means and medians across replications of sample standard deviations of forward rates and the components, in annualized percentage points. Volatilities of forward rates are similar for the DGP and for the estimated series because the models generally fit the cross section of

interest rates well. For risk-neutral rates and term premia, there are substantial differences. Due to the downward bias in the estimated persistence, OLS implies risk-neutral rates that are too stable, with volatilities that are significantly below those of the true risk-neutral rates. On the other hand, bias-corrected estimation leads to estimated risk-neutral rates that are about equally as volatile as for the true model. The volatility of policy expectations is captured better by bias-corrected than conventional estimates. For term premia, the picture is less clear, with OLS premia being slightly too stable and bias-corrected premia too volatile.

To measure the accuracy of estimated rates and premia in relation to the series implied by the true model, we calculate root mean squared errors (RMSEs), in percentage points, for each replication. The last three rows of Table 3 show the means and medians of these RMSEs. Forward rates are naturally fit very accurately, with an average error of about one basis point. Risk-neutral forward rates and forward premia are estimated much more imprecisely, because they depend on the imprecisely measured VAR parameters. Their RMSEs are between 1.2 and 1.4 percentage points. Importantly, the bias-corrected estimates imply lower RMSEs than OLS, indicating that the decomposition of long rates based on these estimates more closely corresponds to the true decomposition.

To assess forecast accuracy, we predict the future 6-month yield for horizons of 1–5 years. We consider random walk (RW) forecasts and model-based forecasts using the OLS and BC estimates. The predictions are made at time  $T$  in each Monte Carlo replication, the forecast errors for each horizon are recorded, and the root mean square forecast errors are calculated across the 1000 Monte Carlo replications. This simulation-based forecast exercise reveals the systematic performance across many samples. In contrast, an assessment of forecast accuracy in a specific interest rate dataset would suffer from the small available sample sizes and the results would be highly sample dependent. Table 4 shows the results. Naturally, RW forecasts are hard to beat, a typical result in this literature, and model-based forecasts mostly have larger RMSEs. However, forecasts based on the BC estimates are more accurate at all horizons than the OLS forecasts.

The evidence from our simulation study clearly demonstrates the value of bias-corrected DTSM estimation for inference about expectations and risk premia. Both for in-sample estimation of risk-neutral rates and term premia, and for out-of-sample forecasts, the BC estimation displays higher accuracy than conventional MLE/OLS estimation.

Table 4. DTSM Monte Carlo study—out-of-sample forecast accuracy

Horizon	RW	OLS	BC
1 year	1.31	1.33	1.24
2 years	2.11	2.42	2.23
3 years	2.77	3.31	3.03
4 years	3.26	4.00	3.68
5 years	3.79	4.68	4.33

NOTE: RMSEs for out-of-sample forecasts of the 6-month yield, across Monte Carlo replications. Competing models: RW, as well as OLS and BC estimates of affine Gaussian DTSM. For details refer to text.

### 5. ESTIMATION OF OVERIDENTIFIED MODELS

We now turn to models that include overidentifying restrictions. After first discussing the type of restrictions that are typically imposed on DTSMs, we propose a bias-corrected estimation procedure for such models. Then, we examine the consequences of bias-corrected estimation for a model with restrictions on risk prices that are common in the DTSM literature.

#### 5.1 Restrictions in DTSMs

Most studies in the DTSM literature impose overidentifying parameter restrictions—either on the dynamic system (Ang and Piazzesi 2003; Kim and Orphanides 2005; Duffee 2011a), on the  $\mathbb{Q}$ -measure parameters (Christensen, Diebold, and Rudebusch 2011), or on the risk sensitivity parameters  $\lambda_0$  and  $\lambda_1$  (Ang and Piazzesi 2003; Kim and Orphanides 2005; Cochrane and Piazzesi 2008; Joslin, Priebsch, and Singleton 2010; Bauer 2011)—with the purpose of avoiding overfit, increasing precision, or facilitating computation. Particularly appealing are restrictions on the risk pricing, that is, on  $(\lambda_0, \lambda_1)$ . Intuitively, under such restrictions, the cross-sectional information helps to pin down the estimates of the VAR parameters. In this way, the no-arbitrage assumption helps overcome problems of small-sample bias and statistical uncertainty. This point was made forcefully by Cochrane and Piazzesi (2008) and has since been used effectively in other studies (Joslin, Priebsch, and Singleton 2010; Bauer 2011).

In the following, we will present a methodology for bias-corrected estimation of restricted DTSMs. This makes it possible to assess the impact of risk price restrictions on small-sample bias. Since a complete analysis of the interaction between bias and various possible risk price restrictions is beyond the scope of this article, we focus on one restricted model with the type of restrictions that are representative in the literature.

#### 5.2 Estimation Methodology

For models with overidentifying restrictions, the estimation methods discussed in Section 4 are not applicable. Here, we introduce an alternative approach based on the framework of HW. They showed that for *any* affine Gaussian DTSM that exactly prices  $N$  linear combinations of yields, all the information in the data can be summarized by the parameters of a reduced-form system—a VAR for the exactly priced linear combinations of yields  $\mathbf{Y}_t^1$  and a contemporaneous regression equation for the linear combinations of yields  $\mathbf{Y}_t^2$  that are priced with error,

$$\mathbf{Y}_t^1 = \boldsymbol{\mu}_1 + \boldsymbol{\Phi}_1 \mathbf{Y}_{t-1}^1 + \mathbf{u}_t^1, \tag{5}$$

$$\mathbf{Y}_t^2 = \boldsymbol{\mu}_2 + \boldsymbol{\Phi}_2 \mathbf{Y}_t^1 + \mathbf{u}_t^2. \tag{6}$$

We denote  $\text{var}(\mathbf{u}_t^1) = \boldsymbol{\Omega}_1$  and  $\text{var}(\mathbf{u}_t^2) = \boldsymbol{\Omega}_2$  (taken to be diagonal). Since we take  $\mathbf{Y}_t^1 = \mathbf{W}\mathbf{Y}_t$  as the risk factors, we have  $\mathbf{Y}_t^1 = \mathbf{X}_t \boldsymbol{\mu}_1 + \boldsymbol{\Phi}_1 \mathbf{Y}_t^1 + \boldsymbol{\Sigma} \boldsymbol{\varepsilon}_t$ , and  $\boldsymbol{\Omega}_1 = \boldsymbol{\Sigma} \boldsymbol{\Sigma}'$ . We also have  $\mathbf{Y}_t^2 = \hat{\mathbf{Y}}_t$ .

HW suggested an efficient two-step procedure for DTSM estimation. In the first step, one obtains estimates of the reduced-form parameters by OLS. In the second step, the structural model parameters are found via minimum-chi-squared estimation: a

chi-squared statistic measures the (weighted) distance between the estimates of the reduced-form parameters and the values implied by the structural parameters, and it is minimized via numerical optimization.

For bias-corrected estimation, we replace the OLS estimates of the VAR in Equation (5) with the bias-corrected parameter estimates. For the contemporaneous regression in Equation (6), OLS is unbiased, so bias correction is not necessary. Having obtained bias-corrected estimates of the reduced-form parameters, we perform the second stage of the estimation as before, minimizing the chi-squared distance statistic. To calculate standard errors for the bias-corrected estimates, we use HW’s asymptotic approximation and simply plug the bias-corrected point estimates into the relevant formula.

#### 5.3 Data and Parameter Estimates

For estimation, we use the zero-coupon yield data described by Gürkaynak, Sack, and Wright (2007). The data are available on the Federal Reserve Board’s website. We use end-of-month observations from January 1985 to December 2011 on yields with maturities of 1, 2, 3, 5, 7, and 10 years.

For the identifying restrictions, we again use the JSZ normalization. Since we want to impose restrictions on risk prices, the model is parameterized in terms of  $(\boldsymbol{\Sigma}\boldsymbol{\lambda}_0, \boldsymbol{\Sigma}\boldsymbol{\lambda}_1, \boldsymbol{\Sigma}, r_\infty^{\mathbb{Q}}, \boldsymbol{\lambda}^{\mathbb{Q}})$  plus the measurement error variance  $\boldsymbol{\Omega}_2$ , which is, as usual, assumed to be diagonal. We focus on  $(\boldsymbol{\Sigma}\boldsymbol{\lambda}_0, \boldsymbol{\Sigma}\boldsymbol{\lambda}_1)$  instead of  $(\boldsymbol{\lambda}_0, \boldsymbol{\lambda}_1)$ , since we do not want our inference to depend on the arbitrary factorization of the covariance matrix of the VAR innovations (Joslin, Priebsch, and Singleton 2010).

To decide which restrictions to impose, we first estimate a maximally flexible model without bias correction. Parameter estimates and standard errors are obtained exactly as in HW. This set of estimates will be called “OLS-UR” (for unrestricted). Then, we set to zero the five elements of  $\boldsymbol{\Sigma}\boldsymbol{\lambda}_1$  with  $t$  statistics less than one. While this is an ad hoc choice of restrictions that ignores issues of the joint significance of parameters and model uncertainty, it is a common practice in the DTSM literature—Bauer (2011) provided a framework to systematically deal with DTSM model selection and model uncertainty. This restricted specification is then estimated in the conventional way (“OLS-R”) as well as using bias correction (“BC-R”).

In Table 5, we report parameter estimates and standard errors for  $(\boldsymbol{\Sigma}\boldsymbol{\lambda}_0, \boldsymbol{\Sigma}\boldsymbol{\lambda}_1, r_\infty^{\mathbb{Q}}, \boldsymbol{\lambda}^{\mathbb{Q}}, \boldsymbol{\Sigma})$ . The  $\mathbb{Q}$ -parameters are very similar across all three sets of estimates, since these are pinned down by the cross section of yields and are largely unaffected by the restrictions. However, the risk price parameters generally change between OLS-R and BC-R. Evidently, even for this tightly restricted model, bias correction has a notable impact on the magnitudes of the estimated risk sensitivities.

#### 5.4 Economic Implications of Bias Correction

We decompose 5- to 10-year forward rates into risk-neutral rate and term premium components. The top panel of Figure 2 displays alternative estimates of the risk-neutral forward rate, while the bottom panel shows estimates of the forward term premium. Both panels also include the actual forward rate. Table 6 presents the summary statistics related to the persistence of the

Table 5. Restricted DTSM—parameter estimates

	OLS-UR			OLS-R			BC-R		
$1200\Sigma\lambda_0$	0.3146 (0.2293)	0.1159 (0.0708)	0.0790 (0.0260)	0.3329 (0.1240)	0.1015 (0.0512)	0.0652 (0.0186)	0.3274 (0.1250)	0.1006 (0.0513)	0.0537 (0.0187)
$\Sigma\lambda_1$	-0.0116 (0.0074)	-0.1303 (0.0439)	-0.0340 (0.2773)	-0.0161 (0.0060)	-0.1229 (0.0358)	0	-0.0092 (0.0060)	-0.1271 (0.0361)	0
	0.0012 (0.0023)	-0.0113 (0.0136)	-0.1577 (0.0856)	0	0	-0.1552 (0.0789)	0	0	-0.1420 (0.0792)
	-0.0008 (0.0008)	-0.0021 (0.0050)	-0.1043 (0.0315)	0	0	-0.1065 (0.0287)	0	0	-0.0886 (0.0288)
$1200r^Q$	15.0635 (0.5418)			15.0609 (0.5420)			15.1111 (0.5456)		
$\lambda^Q$	0.9976 (0.0001)	0.9488 (0.0072)	0.9563 (0.0058)	0.9976 (0.0001)	0.9490 (0.0076)	0.9561 (0.0062)	0.9976 (0.0001)	0.9491 (0.0077)	0.9561 (0.0063)
$1200\Sigma$	0.7264 (0.0284)	0	0	0.7261 (0.0284)	0	0	0.7327 (0.0287)	0	0
	0.0982 (0.0117)	0.2125 (0.0078)	0	0.0981 (0.0117)	0.2125 (0.0078)	0	0.0990 (0.0117)	0.2132 (0.0079)	0
	-0.0291 (0.0044)	0.0211 (0.0040)	0.0728 (0.0029)	-0.0291 (0.0044)	0.0211 (0.0040)	0.0728 (0.0029)	-0.0292 (0.0044)	0.0212 (0.0040)	0.0729 (0.0029)

NOTE: Conventional parameter estimates for maximally flexible model specification (OLS-UR), as well as conventional (OLS-R) and bias-corrected (BC-R) estimates of the model with zero restrictions on risk price parameters.

estimated process, as well as sample standard deviations for the forward rate and its components.

Imposing the restrictions has small effects on the persistence of the estimated process and on the decomposition of long rates. The two series corresponding to OLS-UR and OLS-R in each panel are very close to each other. A look at the summary statistics reveals that the restrictions make the risk-neutral forward rate slightly less volatile and the forward term premium slightly more volatile. The persistence measures indicate a slightly faster speed of mean reversion under the risk price restrictions. Overall, the impact of imposing the five zero restrictions on  $\Sigma\lambda_1$  does not change the implications of the model in economically significant ways.

Bias correcting the DTSM estimates has important economic consequences. The persistence increases significantly, which leads to more variable risk-neutral forward rates. The estimated forward term premium becomes slightly more volatile for BC-R than for OLS-R. Overall, the observations here parallel the ones in the previous section for the JSZ data and model specification. The downward trend in forward rates is attributed to term premia alone for conventional DTSM estimates, whereas

the bias-corrected estimates imply that policy expectations also played an important role for the secular decline. The counter-cyclical pattern of the term premium becomes more pronounced when we correct for bias. With regard to the most recent recession in 2007–2009, the bias-corrected estimates imply a term premium that increases significantly more before and during the economic downturn.

One potential issue with a more persistent VAR process relates to the zero lower bound on nominal interest rates. If the policy rate is close to zero, then forecasts based on a highly persistent VAR can potentially drop below zero and stay negative for an extended period of time. In our setting, at some times during 2010 and 2011, the predicted policy rate becomes negative for horizons up to 2 years. One way to deal with this problem is to truncate the predicted policy rates at zero. Since we focus on distant forward rates, this would not change our results, but it would change the decomposition of other forward rates and yields.

It should be noted that our results are specific to the data, model, and restrictions that we have imposed. They cannot be taken as representative for the impact of risk price restrictions in general. In some cases, restrictions on risk pricing in a DTSM might well be able to largely eliminate small-sample bias (Ball and Torous 1996; Joslin, Priebisch, and Singleton 2010). However, we clearly demonstrate that in a very standard model setting, zeroing out even a majority of the risk price parameters—we set five of the nine parameters in  $\Sigma\lambda_1$  to zero—does not reduce the estimation bias. For both unrestricted and restricted models, small-sample bias is a potentially serious problem. The only way to assess its importance in a particular model and dataset is to obtain bias-corrected estimates, and to evaluate the economic consequences of bias correction. We provide a framework that researchers can use to make an assessment of small-sample bias and its interaction with the parameter restrictions of their choice.

Table 6. Restricted DTSM—summary statistics

	OLS-UR	OLS-R	BC-R
$\max(\text{eig}(\Phi))$	0.9909	0.9904	0.9953
Half-life	64.0	44.0	92.0
IRF at 5 years	0.52	0.41	0.60
$\sigma(f_t^{61,120})$	1.755	1.755	1.755
$\sigma(\tilde{f}_t^{61,120})$	1.148	1.058	1.705
$\sigma(ftp_t^{61,120})$	1.230	1.369	1.425

NOTE: Summary statistics for OLS estimates of unrestricted model (OLS-UR), as well as conventional (OLS-R) and bias-corrected (BC-R) estimates of restricted model. Persistence measures and standard deviations of the fitted 5- to 10-year forward rates and of the corresponding risk-neutral forward rates and forward term premia.



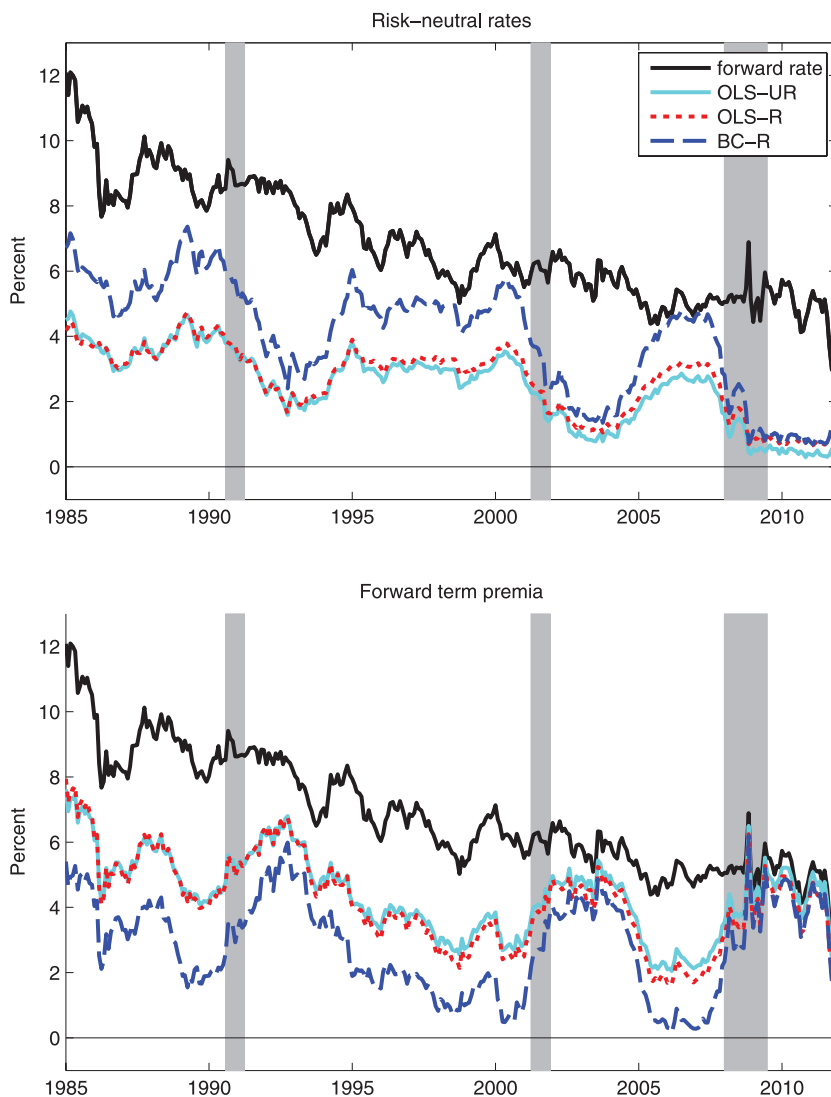


Figure 2. Risk price restrictions—decomposition of forward rates. Decomposition of 5- to 10-year forward rates into risk-neutral and term premium components using alternative DTSM estimates. Sample: monthly observations from January 1985 to December 2011. Gray-shaded areas correspond to NBER recessions. The online version of this figure is in color.

## 6. CONCLUSION

Correcting for finite sample bias in estimates of affine DTSMs has important implications for the estimated persistence of interest rates and for inference about policy expectations and term premia. Risk-neutral rates, which reflect expectations of future monetary policy, show significantly more variation for bias-corrected estimates of the underlying VAR dynamics than for conventional OLS/ML estimates. Our article shows how one can overcome the problem of implausibly stable far-ahead short-rate expectations that several previous studies have criticized. Furthermore, the time series of nominal term premia implied by bias-corrected DTSM estimates show more reasonable variation at business cycle frequencies from a macrofinance perspective than those implied by conventional term premium estimates. Since our results show that correcting for small-sample bias in estimates of DTSMs has important economic implications, researchers and policy makers who analyze movements in interest rates are well advised to use bias-corrected estimators.

Our article is the first to quantify the bias in estimates of DTSMs and opens up several promising directions for future research. In particular, the question of how other methods that aim at improving the specification and/or estimation of the dynamic system fare in terms of bias reduction can be answered using our framework. Among the approaches that have been proposed in the literature are inclusion of survey information (Kim and Orphanides 2005), near-cointegrated specification of the VAR dynamics (Jardet, Monfort, and Pegoraro 2011), and fractional integration (Schotman, Tschernig, and Budek 2008). Furthermore, a thorough investigation of the interactions between risk price restrictions and small-sample bias is warranted.

One issue that this article is not dealing with is whether bias-corrected confidence intervals are more accurate in repeated sampling. A related question is to what extent the reduction of bias increases the variance of the estimator. These are important questions that are beyond the scope of our analysis.

In terms of extensions of our approach, generalizing it to the context of nonaffine and non-Gaussian term structure models

is a desirable next step. One important class of models are affine models that allow for stochastic volatility, which may be spanned or unspanned. Another direction is to develop bias-corrected estimation for models that explicitly impose the zero lower bound on nominal interest rates.

## SUPPLEMENTARY MATERIALS

**Appendix:** Online Appendix (A) explains conventional bootstrap bias convention, (B) details indirect inference bias correction and our algorithm to implement it, (C) explores the performance of alternative bias correction methods in the context of a bivariate VAR, (D) compares alternative bias correction methods for estimating the model of Section 4, and (E) provides details on the parameter bias for the Monte Carlo study in Section 4.4.

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