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THE RESPONSE OF INTEREST RATES TO US AND UK QUANTITATIVE EASING*

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We analyse declines in government bond yields following announcements by the Federal Reserve and the Bank of England of plans to buy longer term debt. Using dynamic term structure models, we decompose US and UK yields into expectations about future short-term interest rates and term premiums. We find that declines in US yields mainly reflected lower expectations of future short-term interest rates, while declines in UK yields appeared to reflect reduced term premiums. Thus, the relative importance of the signalling and portfolio balance channels of quantitative easing may depend on market institutional structures and central bank communication policies.

In late 2008, the Federal Reserve lowered its target policy rate – the overnight federal funds rate – effectively to its zero lower bound. Given a deteriorating outlook for economic growth and a perceived threat of price deflation, the Fed began to purchase longer term securities to push down bond yields and provide additional monetary policy stimulus to the economy. Similarly, in the early spring of 2009, the Bank of England, which had lowered its policy interest rate – the Bank Rate – to its effective zero lower bound, projected weak UK economic growth and a medium-term inflation rate that was below its official 2% target. Therefore, the Bank of England announced plans to purchase government bonds to increase nominal economic activity.

Facing similar circumstances, the Federal Reserve and the Bank of England purchased roughly comparable amounts of bonds – both relative to the size of their economies and to the stocks of outstanding government debt. Recent research also suggests that the two central bank bond purchase programmes induced a comparable reduction in government bond yields in each country. For the US, Gagnon *et al.* (GRRS) (2011) report a cumulative decline in the 10-year US Treasury yield of 91 basis points following eight key announcements about the Fed's first programme of large-scale asset purchases (LSAPs).[1] For the UK, Joyce *et al.* (JLST) (2011) report that long-term UK government bond (or gilt) yields fell a total of about 100 basis points after six key quantitative easing (QE) announcements.[2] Furthermore, both GRRS and JLST provide evidence suggesting that the same mechanism – the portfolio balance channel – was primarily responsible for the bond yield responses in each country.[3] The portfolio balance channel operates when the central bank bond purchases, which change the relative supply of assets held by the

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[1] D'Amico and King (2010), Hamilton and Wu (2012) and Krishnamurthy and Vissing-Jorgensen (2011) provide further discussion.

[2] Note that JLST focus on a two-day event window, while GRRS use a one-day window.

[3] However, as described below, GRRS and JLST emphasise different versions of the portfolio balance channel. GRRS focus on a duration removal version, while JLST focus on a market segmentation version.

private sector, induce equilibrating changes in relative yields. In this channel, announcements of central bank bond purchases push up the prices of the bonds bought and the prices of close substitutes and push down the associated term premiums and yields. The crucial departure from the standard frictionless asset pricing model for a portfolio balance channel is that bonds of different maturities are imperfect substitutes. For example, there may be 'preferred-habitat' investors who have maturity-specific demand for bonds and a less-than-perfect offset to this effect from other 'arbitrageurs' in the market. In this setting of partially segmented markets, the maturity structure of outstanding debt may affect term premiums.

The key alternative mechanism that may account for declines in yields following announcements of future bond purchases is the signalling channel. In the signalling channel, announcements of central bank bond purchases provide information to market participants about current or future economic conditions or monetary policy. For example, the bond purchase announcements could signal a greater commitment to easier monetary policy, so market participants revise down their expectations for future short-term interest rates (assuming, say, a longer period of near-zero policy rates), and longer term yields fall. Therefore, just as the portfolio balance channel is associated with changes in the term premium, the signalling channel is linked to changes in the other components of the standard decomposition of a long-term yield: the average expected level of short-term interest rates over the maturity of the bond.[4]

Still, despite all the similarities in motivation and design, it is not clear that the US and UK bond purchase programmes affected financial markets in the same manner or operated through the same mechanism. One notable puzzle is that the Fed's LSAP and Bank of England's QE announcements had very different effects on overnight index swap (OIS) rates.[5] In the US, long-maturity OIS rates fell nearly in tandem with government yields of a similar maturity, while in the UK, long-maturity OIS rates fell by only a very small portion of the decline in similar maturity UK gilt yields.[6] The different responses of OIS rates in the two countries suggest that different channels for the effects on yields may have been at work. For example, one interpretation of these results is that a signalling channel predominated in the US, so the lower expected short rates tended to lower all long yields equally (including OIS rates), while in the UK, gilts were very imperfect substitutes for swaps, so changes in gilt yields were only imperfectly mirrored in the swaps market.

To shed some light on the various channels through which the US and UK bond purchases may have affected bond prices, we examine the responses of yields in each country using an event study methodology, as in GRRS and JLST. Our event study focuses on results using a dynamic term structure model (DTSM) that can decompose long-term yields into expected short rate and term premium components. With estimated changes in term premiums and monetary policy expectations in each country,

[4] In fact, as discussed below, this usual association is an oversimplification. Shifts in the term premium may alter the expected path of short rates, and news about economic conditions and policy may affect term premiums.

[5] In an OIS, one party pays a fixed interest rate on the notional amount and receives the overnight rate over the entire maturity period. Under absence of arbitrage, OIS rates reflect risk-adjusted expectations of the average policy rate over the horizon corresponding to the maturity of the swap.

[6] See GRRS and JLST and further discussion below.

we can evaluate and compare the responses of yields – and of the components of yields – to US and UK bond purchase announcements.[7]

Our results generally support the differential channels of operation suggested by the responses of OIS rates. That is, our analysis of the US data indicates that more than half of the response of US Treasury yields came from lower expectations for future short-term interest rates. These findings indicate that the magnitude of the portfolio balance effect may not be as large as previously reported. In contrast, our UK results indicate that all of the gilt yield declines on seven key UK QE announcement dates were driven by declines in term premiums. Overall, our contrasting US and UK results suggest that the relative importance of the signalling and portfolio balance effects from central bank bond purchase programmes may depend crucially on specific financial market institutional structures or central bank communication policies. Consequently, managing expectations of future conventional monetary policy through expected short-term rates is likely an important consideration in conducting unconventional monetary policy.

The remainder of this article is organised as follows. Section 1 details our theoretical framework and describes how we extract short-term interest rate expectations and term premiums from bond yields. Section 2 contains our empirical event study analysis of the response of US Treasury yields, while Section 3 contains the comparable analysis for the response of UK gilt yields. Section 4 analyses cross-country yield responses. Finally, Section 5 concludes. An Appendix describes the model estimation.

1. Decomposing Yields with Affine Models

Assessing whether central bank bond purchases affect yields through lower policy expectations or lower term premiums requires an accurate model of expectations for the instantaneous risk-free rate r_t and the term premium. For simplicity, we focus on decomposing $p_t(\tau)$, the price of a zero-coupon bond at time t that has a single payoff, namely \$1, at maturity $t+\tau$. Under standard assumptions (Cochrane, 2001 and the references therein), this price is given by

$$p_t(\tau) = \mathrm{E}_t^P\left(\frac{m_{t+\tau}}{m_t}\right),$$

where the stochastic discount factor, m_t, denotes the value at time t_0 of a claim at a future date t, and the superscript P refers to the actual, or real-world, probability measure underlying the dynamics of m_t.

We follow the usual reduced-form empirical finance approach that models bond prices with unobservable (or latent) factors, here denoted as $\mathbf{x}_t$, and the assumption of no residual arbitrage opportunities.[8] We assume that $\mathbf{x}_t$ follows an affine Gaussian process with constant volatility, with dynamics in continuous time given by the solution to the following stochastic differential equation (SDE):

[7] GRRS, though not JLST, provide a DTSM decomposition. But as we note below, our DTSM decomposition is arguably better suited for this exercise because it produces more accurate short-term interest rate forecasts during the recent sample.

[8] Ultimately, of course, the behaviour of the stochastic discount factor is determined by the preferences of the agents in the economy, as in, for example, Rudebusch and Swanson (2011).

$$\mathrm{d}\mathbf{x}_t = \mathbf{K}^P(\boldsymbol{\theta}^P - \mathbf{x}_t)\mathrm{d}t + \boldsymbol{\Sigma}\mathrm{d}\mathbf{W}_t^P,$$

where $\mathbf{K}^P$ is an $n \times n$ mean-reversion matrix, $\boldsymbol{\theta}^P$ is an $n \times 1$ vector of mean levels, $\boldsymbol{\Sigma}$ is an $n \times n$ volatility matrix and $\mathbf{W}_t^P$ is an n-dimensional Brownian motion. The dynamics of the stochastic discount function are given by

$$\mathrm{d}m_t = r_t m_t \mathrm{d}t + \gamma_t' m_t \mathrm{d}\mathbf{W}_t^P,$$

and the instantaneous risk-free rate, r_t, is assumed affine in the state variables

$$r_t = \delta_0 + \boldsymbol{\delta}_1' \mathbf{x}_t,$$

where $\delta_0 \in R$ and $\boldsymbol{\delta}_1 \in R^n$. The risk premiums, γ_t, are also affine

$$\gamma_t = \gamma_0 + \boldsymbol{\Gamma}_1 \mathbf{x}_t,$$

where $\gamma_0 \in R^n$ and $\boldsymbol{\Gamma}_1 \in R^{n\times n}$.

Duffie and Kan (1996) show that these assumptions imply that zero-coupon yields are also affine in $\mathbf{x}_t$:

$$y_t(\tau) = -\frac{1}{\tau}a(\tau) - \frac{1}{\tau}\mathbf{b}(\tau)'\mathbf{x}_t,$$

where $a(\tau)$ and $\mathbf{b}(\tau)$ are given as solutions to the following system of ordinary differential equations

$$\frac{\mathrm{d}\mathbf{b}(\tau)}{\mathrm{d}\tau} = -\boldsymbol{\delta}_1 - (\mathbf{K}^P + \boldsymbol{\Sigma}\boldsymbol{\Gamma}_1)'\mathbf{b}(\tau), \quad \mathbf{b}(0) = \mathbf{0},$$
$$\frac{\mathrm{d}a(\tau)}{\mathrm{d}\tau} = -\delta_0 + \mathbf{b}(\tau)'(\mathbf{K}^P\boldsymbol{\theta}^P - \boldsymbol{\Sigma}\gamma_0) + \frac{1}{2}\sum_{j=1}^{n}[\boldsymbol{\Sigma}'\mathbf{b}(\tau)\mathbf{b}(\tau)'\Sigma]_{j,j}, \quad a(0) = 0.$$

Thus, the $a(\tau)$ and $\mathbf{b}(\tau)$ functions are calculated *as if* the dynamics of the state variables had a constant drift term equal to $\mathbf{K}^P\boldsymbol{\theta}^P - \boldsymbol{\Sigma}\gamma_0$ instead of the actual $\mathbf{K}^P\boldsymbol{\theta}^P$ and a mean-reversion matrix equal to $\mathbf{K}^P + \boldsymbol{\Sigma}\boldsymbol{\Gamma}_1$ as opposed to the actual $\mathbf{K}^P$.[9] The difference is determined by the risk premium γ_t and reflects investors' aversion to the risks embodied in $\mathbf{x}_t$.

Finally, we define the term premium as

$$TP_t(\tau) = y_t(\tau) - \frac{1}{\tau}\int_t^{t+\tau} \mathrm{E}_t^P(r_s)\mathrm{d}s. \tag{1}$$

That is, the term premium is the difference in expected return between a buy and hold strategy for a τ-year Treasury bond and an instantaneous rollover strategy at the risk-free rate r_t.

2. Analysis of the US Experience

In this Section, we estimate the effect of the Fed's LSAP announcements on expected short-term interest rates and term premiums. We first describe our affine empirical

[9] The probability measure with these alternative dynamics is frequently referred to as the risk-neutral, or Q, probability measure as the expected return on any asset under this measure is equal to the risk-free rate r_t that a risk-neutral investor would demand.

models for US Treasury yields and then provide quantitative results from an event study. However, in light of a potential regime switch in bond pricing following the introduction of a bond purchase programme, the use of these models needs some discussion. Theoretically, we are treating the LSAP or QE announcements as just another series of shocks to the Treasury bond market. As such, there is no notion of a regime switch in terms of the way information is processed and priced into the Treasury yield curve following the purchase announcements. Under that assumption, the models can be used to extract key information about future monetary policy expectations from the variation in the Treasury yield curve.[10]

2.1. *US Empirical Yield Curve Models*

The first model we consider was introduced by Kim and Wright (KW) (2005). This model is estimated on an ongoing basis by the staff of the Federal Reserve Board and was used by GRRS.[11] It is a standard latent three-factor Gaussian term structure model of the kind described in Section 1. The model is estimated using one, two, four, seven and 10-year off-the-run Treasury zero-coupon yields from the Gürkaynak *et al.* (2007) database, as well as three and six-month Treasury bill yields. To facilitate empirical implementation, model estimation includes monthly data on the six and twelve-month-ahead forecasts of the three-month T-bill yield from Blue Chip Financial Forecasts and semi-annual data on the average expected three-month T-bill yield 6–11 years hence from the same source.

The main drawback of this model is one that generally plagues the estimation of any DTSM. Because interest rates are highly persistent, empirical autoregressive models, including DTSMs, suffer from substantial small-sample estimation bias. Specifically, model estimates will generally be biased towards a dynamic system that displays much less persistence than the true process (so estimates of the real-world mean-reversion matrix, $\mathbf{K}^P$, are upward biased). Furthermore, if the degree of interest rate persistence is underestimated, future short rates would be expected to revert to their mean too quickly, causing their expected longer term averages to be too stable. Therefore, the bias in the estimated dynamics distorts the decomposition of yields and contaminates estimates of long-maturity term premia. Bauer *et al.* (2012) provide a complete discussion of the small-sample bias in empirical affine Gaussian DTSMs and simulation-based methods to eliminate it. Here, we construct a DTSM with a number of restrictions imposed both prior to model estimation and based on estimation results that arguably reduce the small-sample estimation bias, partly by imposing a unit-root property on the most persistent factor and partly by using a long sample.[12]

[10] Support for this view is provided in Swanson and Williams (2012). They find that US Treasury yields do not appear to have been constrained in their response to economic news surprises over the 2009–10 period – the focus of our analysis – as compared with their response patterns during the 'normal' period from 1990 to 2000.

[11] The data are available at http://www.federalreserve.gov.

[12] As discussed in Kim and Orphanides (2012), the inclusion of short and long-term survey forecasts of future three-month T-bill rates in the estimation of the KW model serves two purposes. First, it improves the econometric identification of the latent factors, which facilitates model estimation. In our models, we achieve this by imposing the Nelson–Siegel structure described later; see Christensen *et al.* (2011) for a more detailed discussion. Second, the authors argue that it mitigates the upward bias in the estimation of the mean-reversion rates of the state variables described here. Thus, the bias problem is addressed in the KW model, but likely inadequately so, based on our results.

The specific DTSMs we consider are arbitrage-free Nelson–Siegel (AFNS) representations that follow Christensen *et al.* (CDR) (2011) with three state variables, $\mathbf{x}_t = (L_t, S_t, C_t)$.[13] These are described by the following system of SDEs under the risk-neutral Q-measure:[14]

$$\begin{pmatrix} \mathrm{d}L_t \\ \mathrm{d}S_t \\ \mathrm{d}C_t \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda & -\lambda \\ 0 & 0 & \lambda \end{pmatrix} \left[\begin{pmatrix} \theta_1^Q \\ \theta_2^Q \\ \theta_3^Q \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} \right] \mathrm{d}t + \mathbf{\Sigma} \begin{pmatrix} \mathrm{d}W_t^{1,Q} \\ \mathrm{d}W_t^{2,Q} \\ \mathrm{d}W_t^{3,Q} \end{pmatrix}, \quad \lambda > 0.$$

In addition, the instantaneous risk-free rate is defined by

$$r_t = L_t + S_t.$$

CDR show that this specification implies that zero-coupon bond yields are given by

$$y_t(\tau) = L_t + \left(\frac{1 - \mathrm{e}^{-\lambda\tau}}{\lambda\tau} \right) S_t + \left(\frac{1 - \mathrm{e}^{-\lambda\tau}}{\lambda\tau} - \mathrm{e}^{-\lambda\tau} \right) C_t - \frac{a(\tau)}{\tau}, \tag{2}$$

where the factor loadings in the yield function match the level, slope and curvature loadings introduced in Nelson and Siegel (1987). The final yield-adjustment term, $a(\tau)/\tau$, captures convexity effects due to Jensen's inequality.[15]

The maximally flexible specification of the AFNS model has P-dynamics given by[16]

$$\begin{pmatrix} \mathrm{d}L_t \\ \mathrm{d}S_t \\ \mathrm{d}C_t \end{pmatrix} = \begin{pmatrix} \kappa_{11}^P & \kappa_{12}^P & \kappa_{13}^P \\ \kappa_{21}^P & \kappa_{22}^P & \kappa_{23}^P \\ \kappa_{31}^P & \kappa_{32}^P & \kappa_{33}^P \end{pmatrix} \left[\begin{pmatrix} \theta_1^P \\ \theta_2^P \\ \theta_3^P \end{pmatrix} - \begin{pmatrix} L_t \\ S_t \\ C_t \end{pmatrix} \right] \mathrm{d}t + \begin{pmatrix} \sigma_{11} & 0 & 0 \\ \sigma_{21} & \sigma_{22} & 0 \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} \begin{pmatrix} \mathrm{d}W_t^{1,P} \\ \mathrm{d}W_t^{2,P} \\ \mathrm{d}W_t^{3,P} \end{pmatrix}. \tag{3}$$

We estimate our AFNS models using the same daily nominal US Treasury zero-coupon yields used in the estimation of the KW model.[17] The data run from December 1, 1987, until December 31, 2010, for eight maturities: three months, six months, one year, two years, three years, five years, seven years and 10 years.

To select the best fitting specification of the AFNS model's real-world dynamics, we first build on the findings in CDR and limit the $\mathbf{\Sigma}$ volatility matrix to be diagonal. Then, to determine the appropriate specification of the mean-reversion matrix $\mathbf{K}^P$, we use a general-to-specific modelling strategy that restricts the least significant parameter in the estimation to zero and then re-estimates the model. This strategy of eliminating the

[13] For related applications of the AFNS model, see Christensen *et al.* (2010), who examine yields for nominal and real Treasuries, and Christensen *et al.* (2009), who examine short-term LIBOR and highly rated financial firms' corporate bond rates.

[14] As discussed in CDR, with a unit root in the level factor under the pricing measure, the model is not arbitrage-free with an unbounded horizon; therefore, as is often done in theoretical discussions, we impose an arbitrary maximum horizon. Also, following CDR, we identify this class of models by fixing the θ^Q means under the Q-measure at zero without loss of generality.

[15] The model is completed with a risk premium specification that connects the factor dynamics to the dynamics under the real-world P-measure. It is important to note that there are no restrictions on the dynamic drift components under the empirical P-measure beyond the requirement of constant volatility. To facilitate empirical implementation, we use the essentially affine risk premium introduced in Duffee (2002).

[16] As noted in CDR, the unconstrained AFNS model has a sign restriction and three parameters less than the standard canonical three-factor Gaussian DTSM.

[17] The Appendix provides details of our estimation methodology.

least significant coefficients is carried out down to the most parsimonious specification, which has a diagonal $\mathbf{K}^P$ matrix. The final specification choice is based on the values of the Akaike and Bayes information criteria (AIC and BIC) as per Christensen *et al.* (2010).[18] The summary statistics of the model selection process are reported in Table 1. Both information criteria are minimised by specification (5), which has a $\mathbf{K}^P$ matrix specified as

$$\mathbf{K}_{US}^P = \begin{pmatrix} \kappa_{11}^P & 0 & 0 \\ \kappa_{21}^P & \kappa_{22}^P & \kappa_{23}^P \\ 0 & 0 & \kappa_{33}^P \end{pmatrix}.$$

Finally, to mitigate the small-sample bias problem in the estimation of the parameters in $\mathbf{K}^P$, we impose a unit-root property on the Nelson–Siegel level factor. Thus, in the end, our preferred specification of the AFNS model for the US has P-dynamics given by[19]

$$\begin{pmatrix} \mathrm{d}L_t^{US} \\ \mathrm{d}S_t^{US} \\ \mathrm{d}C_t^{US} \end{pmatrix} = \begin{pmatrix} 10^{-7} & 0 & 0 \\ \kappa_{21}^P & \kappa_{22}^P & \kappa_{23}^P \\ 0 & 0 & \kappa_{33}^P \end{pmatrix} \left[\begin{pmatrix} 0 \\ \theta_2^P \\ \theta_3^P \end{pmatrix} - \begin{pmatrix} L_t^{US} \\ S_t^{US} \\ C_t^{US} \end{pmatrix} \right] \mathrm{d}t + \begin{pmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} \mathrm{d}W_t^{1,P} \\ \mathrm{d}W_t^{2,P} \\ \mathrm{d}W_t^{3,P} \end{pmatrix}.$$

There are two things to note regarding this specification. First, the Nelson–Siegel level factor is restricted to be an independent unit-root process under both probability measures.[20] As discussed below, this restriction helps improve forecast performance

Table 1

Evaluation of Alternative Specifications of the AFNS Model of US Treasury Yields

Alternative Specifications	Goodness-of-fit statistics				
	log L	k	p-value	AIC	BIC
(1) Unrestricted $\mathbf{K}^P$	280,690	24	n.a.	−561,332	−561,172
(2) $\kappa_{32}^P = 0$	280,690	23	0.6547	−561,334	−561,181
(3) $\kappa_{32}^P = \kappa_{31}^P = 0$	280,690	22	0.5271	−561,336	−561,189
(4) $\kappa_{32}^P = \kappa_{31}^P = \kappa_{12}^P = 0$	280,689	21	0.2367	−561,336	−561,196
(5) $\kappa_{32}^P = \ldots = \kappa_{13}^P = 0$	280,689	20	0.5271	*−561,338*	*−561,205*
(6) $\kappa_{32}^P = \ldots = \kappa_{21}^P = 0$	280,685	19	0.0034	−561,331	−561,205
(7) $\kappa_{32}^P = \ldots = \kappa_{23}^P = 0$	280,665	18	0.0000	−561,293	−561,174

Notes. There are seven alternative estimated specifications of the AFNS model of US Treasury yields with the unrestricted 3-by-3 $\mathbf{K}^P$ matrix being the most flexible. Each specification is listed with its maximum log likelihood value (log L), number of parameters (k), the p-value from a likelihood ratio test of the hypothesis that it differs from the specification above with one more free parameter, and the Akaike information criterion (AIC) and Bayes information criterion (BIC), whose minimum values are given in italics. The sample is daily from December 1, 1987, to December 31, 2010, a total of 5,757 observations.

[18] See Harvey (1989) for further details.

[19] The simple dynamic three-factor Gaussian model introduced in Duffee (2011) is qualitatively close to our preferred model (it has $\kappa_{21}^P = 0$, but $\kappa_{32}^P \neq 0$). Duffee reports good forecast performance for this model, but uses a sample of US Treasury yields that differs from ours. Furthermore, his state variables are identical to the three first principal components, whereas our state variables are the filtered AFNS factors, which are *not* identical to the three first principal components.

[20] Due to the unit-root property of the first factor, we can arbitrarily fix its mean at $\theta_1^P = 0$.

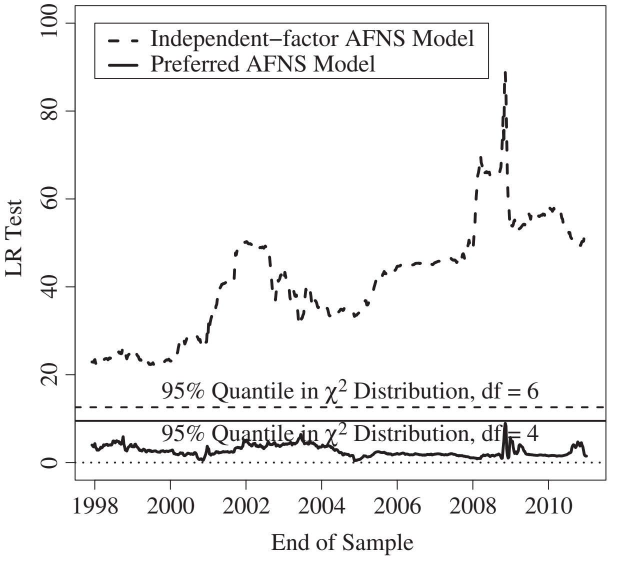

Fig. 1. *Likelihood Ratio Tests of Parameter Restrictions in US AFNS Models*
Notes. Illustration of the value of likelihood ratio tests of the restrictions imposed in the independent-factor and preferred AFNS models relative to the AFNS model with unrestricted $\mathbf{K}^P$ matrix and diagonal $\mathbf{\Sigma}$ matrix. The analysis covers weekly re-estimations of expanding samples from December 4, 1998, to December 31, 2010, a total of 683 observations, while the full data set is weekly covering the period from December 4, 1987, to December 31, 2010. The 95 percentiles in the relevant χ^2 distributions are shown with horizontal lines.

independent of the specification of the remaining elements of $\mathbf{K}^P$.[21] Second, we test the significance of the four parameter restrictions imposed on $\mathbf{K}^P$ in the preferred AFNS model relative to the corresponding AFNS model with an unrestricted $\mathbf{K}^P$ matrix.[22] As shown in Figure 1, the four parameter restrictions are statistically insignificant throughout our sample period. Thus, our preferred AFNS model is flexible enough to capture the relevant information in the data. To assess the robustness of our results, we also consider both the unconstrained AFNS model described in Equation (3), which is the AFNS model closest to the canonical Gaussian $A_0(3)$ model of Dai and Singleton (2000), and the independent-factor AFNS model favoured by CDR, even though likelihood ratio tests of its parameter restrictions (also shown in Figure 1) indicate that it is too parsimonious.[23]

To study bond investors' expectations in real time, we perform a rolling re-estimation of the models on expanding samples – adding one day of observations each time, a

[21] As described in detail in Bauer *et al.* (2012), bias-corrected $\mathbf{K}^P$ estimates are typically very close to a unit-root process, so we view the imposition of the unit-root restriction as a simple shortcut to overcome small-sample estimation bias.

[22] That is, we test the hypotheses $\kappa^P_{12} = \kappa^P_{13} = \kappa^P_{31} = \kappa^P_{32} = 0$ jointly using a standard likelihood ratio test.

[23] In unreported results, (*i*) we repeated the forecast exercise in Diebold and Li (2006), (*ii*) we estimated all eight admissible specifications of two-factor AFNS models (i.e. those with only a level and a slope factor) with and without unit-root properties imposed and (*iii*) we studied more flexible specifications of the volatility matrix within the AFNS model. For both the US and UK samples, none of these alternatives systematically outperformed our preferred specifications.

total of 3,249 estimations. As a result, the end dates of the expanding samples run from January 2, 1998, to December 31, 2010. For each end date during that period, we calculate the average expected path for the overnight rate, $(1/\tau)\int_t^{t+\tau} \mathrm{E}_t^P(r_s^{US})\mathrm{d}s$, as well as the associated term premium – assuming the two components sum to the fitted bond yield, $\hat{y}_t^{US}(\tau)$. Importantly, the estimates of these two components rely essentially only on information that was available in real time.

2.2. *Comparison of US DTSMs*

For a start, we use our AFNS models and the KW model to forecast the target overnight federal funds rate one and two years ahead at the end of each month over the period from January 2, 1998, until December 31, 2010. The summary statistics for the forecast errors relative to the subsequent realisations of the target overnight federal funds rate set by the Federal Open Market Committee (FOMC) are reported in Table 2, which also contains the forecast errors obtained using a random walk assumption. We note the weaker forecast performance of the KW model relative to our preferred AFNS model. Figure 2 compares the forecasts at the two-year horizon from the KW model and the preferred AFNS model to the subsequent target rate realisations. The KW model systematically overestimates the subsequent target rate realisations since the fall of 2008, which is the period of interest for studying the effects of the LSAP programme.

Figure 3 shows the time series of the 10-year term premium from our preferred AFNS model and the 10-year term premium from the KW model. Over the whole sample, the KW term premium averages about half the size of that produced by the AFNS model. The two measures of the term premium have a high degree of correlation (almost 60%) but also exhibit important differences at the low points of the monetary policy cycles – notably, during 2002–4 and 2008 to the present. During these periods, the KW premium is very low and indeed appears to turn negative in the fall of 2010.

These low term premiums may be an artefact of the model estimation bias noted above. Any bias in the model-generated expectations for future short-term interest rates will translate one-for-one into a similar bias, but with opposite sign, in the estimated term premiums. Specifically, at low points in the interest rate cycle, it appears that the

Table 2

Summary Statistics for Target Federal Funds Rate Forecast Errors

	One-year forecast		Two-year forecast	
Forecasting method	Mean	RMSE	Mean	RMSE
Random walk	40.03	170.18	84.12	282.21
Kim & Wright model	57.05	142.14	140.93	252.58
Unconstrained AFNS model	6.05	161.24	33.78	263.92
Indep.-factor AFNS model	32.20	158.28	70.22	263.80
Preferred AFNS model	9.61	136.68	70.85	250.32

Notes. Summary statistics of the forecast errors – mean and root mean-squared errors (RMSEs) – of the target overnight federal funds rate one and two years ahead. The forecasts are monthly starting on January 31, 1998, and running until December 31, 2010, for the one-year forecasts (156 forecasts), and until December 31, 2009, for the two-year forecasts (144 forecasts). All measurements are expressed in basis points.

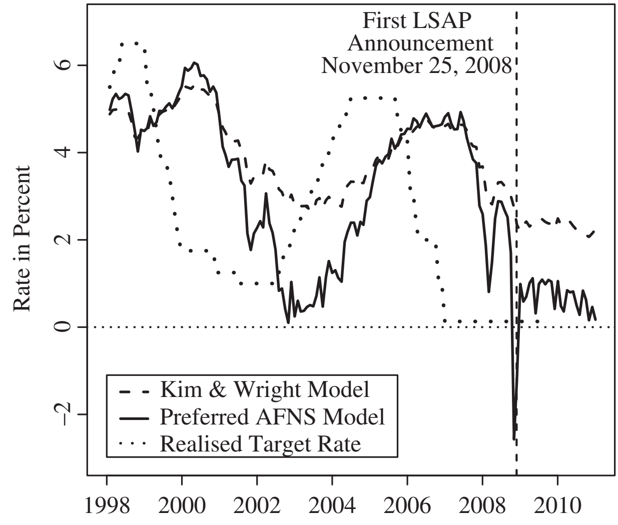

Fig. 2. *Forecasts of the Target Overnight Federal Funds Rate*
Notes. Forecasts of the target overnight federal funds rate two years ahead from the preferred AFNS model and the Kim and Wright model. Subsequent realisations of the target overnight federal funds rate are included, so at date *t*, the figure shows forecasts as of time *t* and the realisation from *t* plus two years. The forecast data are end-of-month observations from January 31, 1998, to December 31, 2010.

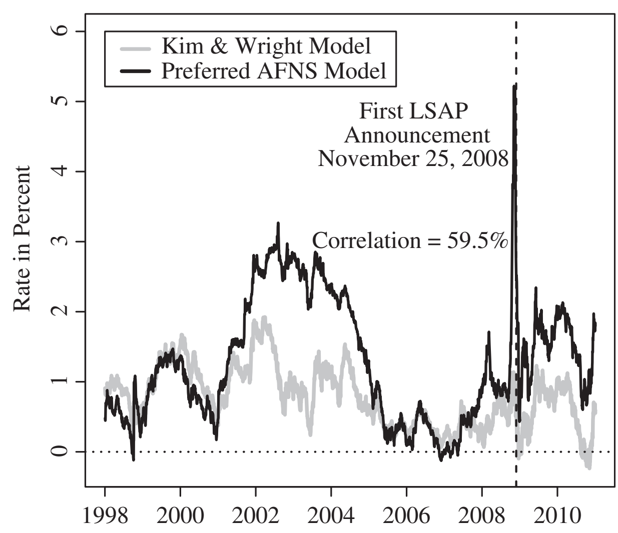

Fig. 3. *Two Estimates of 10-Year US Term Premiums*
Notes. The Figure illustrates the 10-year US Treasury zero-coupon term premium estimates from the preferred AFNS model as well as the corresponding estimates from the KW model. Both series are daily, covering the period from January 2, 1998, to December 31, 2010.

KW model generates expectations for future short-term rates that are too high – that is, a quicker reversion to mean – and, equivalently, estimates of term premiums that appear too low. This potential inaccuracy raises doubts about the KW term premium decomposition exactly during the crucial period since late November 2008 that we are interested in analysing.

2.3. *Response of US Yields to Bond Purchase Announcements*

We analyse the effects of the Federal Reserve's bond purchases using an event study methodology that examines changes in US interest rates over one-day intervals around announcements of future bond purchases. Of course, though widely used, this is an imperfect technique. We have no reliable measures of what was expected prior to each Fed announcement, so, following GRRS, we assume that the entire announcement was a complete surprise. This is likely to underestimate the interest rate response as, especially for the later announcements, market participants may have anticipated some Fed action.[24] Also, a one-day event window may be too short to capture all of the announcements' effects – again, perhaps biasing downward the estimated size of these effects.[25] On the other hand, a one-day window may capture an exaggerated initial market response that is unwound over time as market makers and investors adjust. Finally, even during a one-day window, other news may have been released that significantly affected interest rates and obscured the effects we are trying to assess. Although we believe that a majority of the interest rate movements we examine reflected new information from the Fed's announcements, at the very least our results provide a careful comparison to the well-known GRRS results, using different empirical models to extract term premiums.

We start our analysis with a model-free inspection of the data. The eight key announcements regarding the Federal Reserve's first LSAP programme highlighted by GRRS are listed in Table 3. Table 4 shows the changes on these dates in five of the eight yield maturities we use in model estimation. Five- and 10-year US Treasury yields declined almost 100 basis points over the eight announcement dates, while shorter maturities changed much less. However, without a dynamic modelling of the entire yield curve, it is not possible to conclude whether short-term interest rate expectations or changes in term premiums drive observed yield changes.

To first get a sense of how widely these Treasury yield reactions were shared in other markets, we analyse the reaction of OIS rates.[26] These rates represent average expectations for the effective federal funds rate over the given maturity. Of course, as for any financial claim, OIS rates contain their own set of risk premiums and are not pure measures of future policy expectations. Still, the very small net changes in Treasury–OIS spreads implied by the responses reported in Tables 4 and 5 suggest that there was a common factor behind the observed declines in Treasury yields and OIS rates. This common factor may have been a shift in the expectations component in long-term interest rates. Alternatively, the rates may have shared a shift in risk

[24] We only examine the first round of Fed purchases because the information releases regarding the second round of purchases and the subsequent maturity extension programme were more diffuse and less amenable to an event study analysis.

[25] JLST, for example, use a two-day window in their UK analysis.

[26] Krishnamurthy and Vissing-Jorgensen (2011) use federal funds futures with maturities up to 24 months and extrapolate beyond that to measure the response of long-term monetary policy expectations. Their method suggests an upper bound of 40 basis points for the decline in policy expectations.

Table 3

Key Federal Reserve LSAP Announcements

No.	Date	Event	Description
I	November 25, 2008	Initial LSAP announcement	Fed announces purchases of $100 billion in GSE debt and up to $500 billion in mortgage-backed securities (MBS).
II	December 1, 2008	Bernanke speech	Chairman Bernanke indicates that the Fed could purchase long-term Treasury securities.
III	December 16, 2008	FOMC statement	The first FOMC statement that mentions possible purchases of long-term Treasuries.
IV	January 28, 2009	FOMC statement	FOMC states that it is ready to expand agency debt and MBS purchases and to purchase long-term Treasuries.
V	March 18, 2009	FOMC statement	Fed will purchase an additional $750 billion in agency MBS and $100 billion in agency debt. Also, it will purchase $300 billion in long-term Treasury securities.
VI	August 12, 2009	FOMC statement	Fed is set to slow the pace of the LSAP. The final purchases of Treasury securities will be in the end of October instead of mid-September.
VII	September 23, 2009	FOMC statement	Fed's purchases of agency debt and MBS will end in the first quarter of 2010, while its Treasury purchases will end as planned in October.
VIII	November 4, 2009	FOMC statement	Amount of agency debt capped at $175 billion instead of the $200 billion previously announced.

Table 4

Changes in US Treasury Yields on LSAP Announcement Dates

	Maturity				
Event	6-month	1-year	2-year	5-year	10-year
I November 25, 2008	−5	−9	−14	−22	−21
II December 1, 2008	−3	−6	−12	−21	−22
III December 16, 2008	−7	−8	−11	−16	−17
IV January 28, 2009	−5	−1	5	10	12
V March 18, 2009	−13	−17	−26	−47	−52
VI August 12, 2009	1	0	−1	1	6
VII September 23, 2009	−1	−2	−4	−4	−2
VIII November 4, 2009	−1	−1	−1	3	7
Total net change	−34	−45	−65	−97	−89

Notes. Changes are measured in basis points.

premiums following the LSAP announcements. For example, GRRS argue that the announced changes in the supply of long-term bonds affect the aggregate amount of duration available in the market and the pricing of the associated interest rate risk term premium, which is shared by all similar-duration bonds. In their duration removal version of the portfolio balance channel, lowering aggregate duration risk can reduce term premiums in all fixed-income securities.

A second set of fixed-income securities that investors could view as relatively close substitutes to US Treasuries are US corporate bonds. Table 6 contains the yield changes

Table 5

Changes in US OIS Rates on LSAP Announcement Dates

	Maturity				
Event	6-month	1-year	2-year	5-year	10-year
I November 25, 2008	−5	−7	−14	−25	−28
II December 1, 2008	−5	−5	−13	−21	−19
III December 16, 2008	−17	−17	−15	−29	−32
IV January 28, 2009	0	4	6	11	14
V March 18, 2009	−3	−5	−12	−27	−38
VI August 12, 2009	−2	−2	−1	−2	1
VII September 23, 2009	−2	−3	−5	−6	−5
VIII November 4, 2009	−1	−2	−3	1	5
Total net change	−35	−37	−58	−97	−102

Notes. Changes are measured in basis points.

for three rating categories (AA, BBB, B) across five maturities.[27] Corporate bond yields generally declined but by less than Treasury yields. Again, a common factor seems to be present, although likely tempered by some negative news on announcement dates regarding the economic outlook so that lower credit quality bonds faced greater perceived risk of default, especially in the near term. Thus, credit spreads increased, net, for all three rating categories but increased more the lower the credit quality and the shorter the maturity of the bond.

A third important segment of the US fixed-income markets that could serve as a close substitute for US Treasury bond investors is the huge market for interest rate swaps tied to the US dollar London interbank offered rate (LIBOR). The reaction in this market to the LSAP announcements is reported in Table 7, where we note that both LIBOR and swap rates declined in tandem with Treasury yields and OIS rates.

The fairly similar reaction of US Treasury yields, OIS rates, corporate bond yields, and swap interest rates to LSAP announcements provides little evidence of pronounced market segmentation in the US fixed-income market during this period or a simple market segmentation version of the portfolio balance channel. The model-free evidence is consistent with a view that US LSAP announcements mainly worked through the signalling channel, whereby long-term interest rates were depressed as purchase announcements in essence indicated that short-term interest rates would be low for longer than previously anticipated. Alternatively, the model-free evidence could also be seen as consistent with the GRRS duration removal version of the portfolio balance channel, in which the market price of duration risk increases with Fed purchase announcements, and the term premiums on all fixed-income securities of a long maturity fall. To distinguish between these last two channels, we use the empirical DTSMs described in the previous Section. Using these models, we decompose the response of Treasury yields to the LSAP announcements into three components:

(*i*) the response of the estimated average target federal funds rate until maturity;

[27] See Christensen and Lopez (2008) for a description of the corporate bond data, which are obtained from Bloomberg.

Table 6

Changes in US Corporate Bond Yields on LSAP Announcement Dates

Event	6-month	1-year	2-year	5-year	10-year
AA-rated US industrial corporate bonds					
I November 25, 2008	6	1	−6	−18	−24
II December 1, 2008	−13	−13	−12	−24	−23
III December 16, 2008	10	6	0	−16	−23
IV January 28, 2009	−2	0	5	11	13
V March 18, 2009	−5	−13	−22	−41	−49
VI August 12, 2009	−2	−1	−2	2	7
VII September 23, 2009	−1	−1	−3	−4	−2
VIII November 4, 2009	1	−1	0	6	14
Total net change	−5	−21	−40	−85	−89
Event	6-month	1-year	2-year	5-year	10-year
BBB-rated US industrial corporate bonds					
I November 25, 2008	8	2	−4	−17	−23
II December 1, 2008	−4	−5	−3	−16	−14
III December 16, 2008	−3	−7	−13	−14	−22
IV January 28, 2009	−4	−2	2	8	10
V March 18, 2009	−2	−10	−19	−39	−45
VI August 12, 2009	−2	−1	−2	1	5
VII September 23, 2009	−1	−1	−3	−4	−2
VIII November 4, 2009	−1	−4	−3	4	11
Total net change	−11	−27	−45	−77	−80
Event	6-month	1-year	2-year	5-year	10-year
B-rated US industrial corporate bonds					
I November 25, 2008	41	34	27	14	9
II December 1, 2008	0	−1	0	−13	−11
III December 16, 2008	1	−3	−9	−21	−29
IV January 28, 2009	2	4	9	15	17
V March 18, 2009	4	−4	−20	−32	−40
VI August 12, 2009	−8	−7	−7	−5	1
VII September 23, 2009	−8	−8	−10	−11	−9
VIII November 4, 2009	5	3	4	10	19
Total net change	36	18	−8	−42	−43

Notes. Changes in US industrial corporate bond yields across three rating categories (AA, BBB and B) measured in basis points. The data are from Bloomberg.

(*ii*) the response of the term premium defined as the difference between the model fitted Treasury yield and the average expected target rate and
(*iii*) a residual that reflects variation not accounted for by the model.

The results of the decomposition of the response of the 10-year US Treasury yield on the eight LSAP announcement dates are reported in Table 8. First, we note the fairly large variation in the decompositions across the four models, which is a reflection of the inherent uncertainty in this type of analysis.[28] Second, it is worth highlighting the

[28] This uncertainty is also highlighted by the wide confidence intervals estimated in Bauer and Rudebusch (2011).

Table 7

Changes in US LIBOR and Swap Rates on LSAP Announcement Dates

Event	Maturity			
	3-month	2-year	5-year	10-year
I November 25, 2008	1	−17	−29	−29
II December 1, 2008	−1	−8	−18	−17
III December 16, 2008	−29	−26	−34	−32
IV January 28, 2009	−1	4	11	14
V March 18, 2009	−7	−25	−33	−39
VI August 12, 2009	−1	−4	−3	1
VII September 23, 2009	0	−6	−6	−5
VIII November 4, 2009	0	−3	2	5
Total net change	−39	−86	−109	−101

Notes. Changes are measured in basis points. Note that the response of the three-month US LIBOR uses a two-day window as it is determined daily around 11 a.m. GMT, well before the release of any of the US LSAP announcements.

qualitative agreement of the models regarding the response on the first five LSAP announcements, for example, they all suggest that policy expectations declined on four of these five dates. However, the magnitudes vary, and as explained previously, a model with a big response in the policy expectations component relative to another model will show an equally smaller response in the term premium component as the observed yield change is the same for all models. Third, in terms of the KW model, we replicate the result of GRRS, whose emphasis on the portfolio balance channel is based on the observation that about 80% of the decline in the 10-year US Treasury yield to the LSAP announcements is explained by declines in the term premium.[29] Importantly, though, the definition of the term premium in (1) requires a conditional forecast of future short rates and as, based on Table 2, our preferred AFNS model has delivered the most accurate real-time forecasts of future short rates in the past, we choose to focus on that model in the remainder of this Section. Thus, using the preferred AFNS model, one key result is that the cumulative net decline in the expectations component of the 10-year yield fell by 53 basis points. That is, almost 60% of the total change in the 10-year Treasury yield following the eight LSAP announcements is explained by declines in the expected future target rates. Declines in term premiums only account for about a third of the yield change (29 basis points). Similarly, Bauer and Rudebusch (2011), using a bias-corrected estimate of the term premium, report that about 50% of the 10-year yield change is accounted for by changes in the expectations component.

If we analyse the model-based decomposition from the preferred AFNS model in greater detail, we find that, for the initial four dates with LSAP-related announcements (November 25, 2008; December 1, 2008; December 16, 2008; and January 28, 2009), a large part of the change in the observed yield curve is assigned by the model to shifts in expectations about future monetary policy rather than in term premiums. Of the total decrease of 48 basis points in the 10-year yield on these four dates, 31 basis points are

[29] Note that this is an extreme interpretation of the decomposition from the KW model as the changes in policy expectations are minimised by the unexplained residual. At the other extreme, there would be a 35/65 split between the expectations and term premium components.

Table 8

Decomposition of Responses of 10-year US Treasury Yield

		Decomposition from models			
Event	Model	Avg. target rate next 10 years	10-year term premium	Residual	10-year Treasury yield
I November 25, 2008	Kim & Wright	−7	−17	3	−21
	Unconstrained AFNS	−17	−7	3	
	Indep.-factor AFNS	−2	−17	−2	
	Preferred AFNS	−20	0	−2	
II December 1, 2008	Kim & Wright	−7	−17	2	−22
	Unconstrained AFNS	−23	−2	3	
	Indep.-factor AFNS	−1	−19	−2	
	Preferred AFNS	−10	−10	−2	
III December 16, 2008	Kim & Wright	−7	−12	1	−17
	Unconstrained AFNS	−22	3	2	
	Indep.-factor AFNS	−1	−13	−3	
	Preferred AFNS	−7	−7	−3	
IV January 28, 2009	Kim & Wright	3	9	0	12
	Unconstrained AFNS	5	6	1	
	Indep.-factor AFNS	−7	14	5	
	Preferred AFNS	6	1	5	
V March 18, 2009	Kim & Wright	−16	−40	4	−52
	Unconstrained AFNS	−54	−5	7	
	Indep.-factor AFNS	−11	−27	−15	
	Preferred AFNS	−14	−23	−15	
VI August 12, 2009	Kim & Wright	1	3	2	6
	Unconstrained AFNS	7	−1	1	
	Indep.-factor AFNS	3	−3	6	
	Preferred AFNS	−1	1	6	
VII September 23, 2009	Kim & Wright	−1	−1	0	−2
	Unconstrained AFNS	−3	2	0	
	Indep.-factor AFNS	2	−5	1	
	Preferred AFNS	−5	2	1	
VIII November 4, 2009	Kim & Wright	2	5	0	7
	Unconstrained AFNS	3	4	−1	
	Indep.-factor AFNS	−1	6	3	
	Preferred AFNS	−1	5	3	
Total net change	Kim & Wright	−31	−71	13	−89
	Unconstrained AFNS	−104	0	16	
	Indep.-factor AFNS	−18	−64	−7	
	Preferred AFNS	−53	−29	−7	

Notes. The decomposition of responses of the 10-year US Treasury yield on eight LSAP announcement dates into changes in (*i*) the average expected target rate over the next 10 years, (*ii*) the 10-year term premium, and (*iii*) the unexplained residual based on empirical DTSMs of US Treasury yields. All changes are measured in basis points.

explained by declines in policy expectations, while declines in term premiums only account for 16 basis points. Thus, two-thirds of the decrease can be attributed to declines in expectations about future monetary policy on these dates. For the LSAP announcement on March 18, 2009, in which Treasury security purchases were

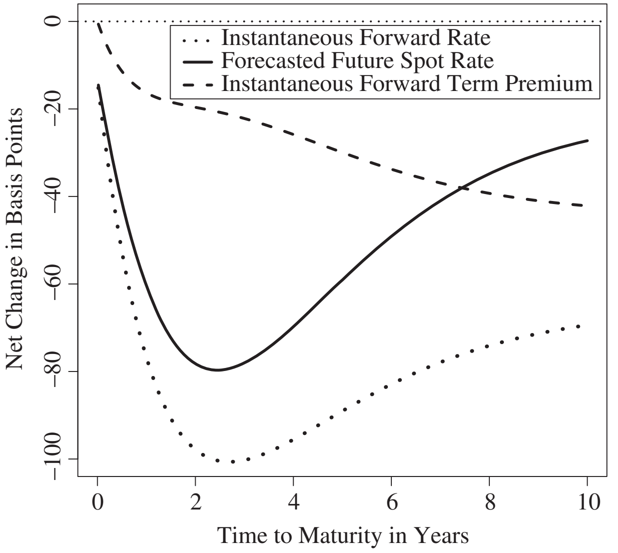

Fig. 4. *Decomposition of Net Response of US Treasury Forward Rates*
Notes. Illustration of the decomposition of the net response of instantaneous US Treasury forward rates to eight LSAP announcements into (*i*) forecasted future instantaneous spot rates and (*ii*) instantaneous forward term premiums based on the preferred AFNS model of US Treasury yields.

introduced, the main part of the decline in bond yields was driven by declines in the term premiums rather than declines in policy expectations. As a consequence, we also see significant declines in the Treasury–OIS spread at all maturities on this day. Finally, for the remaining three LSAP announcement dates, the responses were more modest. Short-term interest rate expectations edged down, while term premiums edged higher with the perceived end to the Fed's Treasury bond purchases. Consistent with these results, there is a uniform increase in the Treasury–OIS spread in the one to 10-year maturity range on all three announcement dates.

Our preferred AFNS model also allows us to study the response of forward rates. The net response of the fitted forward rate curve as well as its decomposition into forecasted future instantaneous spot rates and instantaneous forward term premiums is shown in Figure 4. Not surprisingly, policy expectations in the medium term – two–three years ahead – reacted the most to the LSAP announcements, while the 10-year-ahead spot rate expectations declined by a smaller amount (a total of 27 basis points). Term premiums declined at all horizons but more so at the long end of the yield curve.

To summarise our findings for the US, the key conclusion is that changes in policy expectations appear to have played an important role in the reaction of US Treasury yields on the key announcement dates in the Fed's first LSAP programme.

3. Analysis of the UK Experience

In this Section, we estimate the effect of the Bank of England's QE announcements on expected short-term interest rates and term premiums. We first describe our empirical affine models for UK gilt yields and then provide quantitative results from an event study.

3.1. *UK Empirical Yield Curve Models*

Again, we construct a preferred DTSM with restrictions that arguably reduce the small-sample estimation bias. Our specific AFNS models are estimated using daily data on UK zero-coupon gilt yields with the same eight maturities used in the US analysis: three months, six months, one year, two years, three years, five years, seven years, and 10 years.[30]

To select the best fitting specification of the AFNS model's real-world dynamics, we proceed as in the US analysis, that is, we first limit the $\mathbf{\Sigma}$ volatility matrix to be diagonal. Second, we use a general-to-specific modelling strategy to determine the appropriate specification of the mean-reversion matrix $\mathbf{K}^P$ where the least significant parameter is eliminated in each step. As before, the final specification choice is based on the values of the AIC and BIC. The summary statistics of the model selection process are reported in Table 9. The AIC is minimised by specification (6) within Table 9, which has a $\mathbf{K}^P$ matrix given by

$$\mathbf{K}^P_{UK} = \begin{pmatrix} \kappa^P_{11} & 0 & 0 \\ 0 & \kappa^P_{22} & \kappa^P_{23} \\ 0 & 0 & \kappa^P_{33} \end{pmatrix},$$

while the BIC calls for an even more parsimonious, diagonal specification of $\mathbf{K}^P$. In light of the individual significance of the marginal parameter, κ^P_{23}, we choose specification (6) as our preferred one.

Finally, to mitigate the small-sample bias problem in the estimation of the parameters in $\mathbf{K}^P$, we impose a unit-root property on the Nelson–Siegel level factor. Thus, in the end, our preferred specification of the AFNS model for the UK has P-dynamics given by

$$\begin{pmatrix} \mathrm{d}L^{UK}_t \\ \mathrm{d}S^{UK}_t \\ \mathrm{d}C^{UK}_t \end{pmatrix} = \begin{pmatrix} 10^{-7} & 0 & 0 \\ 0 & \kappa^P_{22} & \kappa^P_{23} \\ 0 & 0 & \kappa^P_{33} \end{pmatrix} \left[\begin{pmatrix} 0 \\ \theta^P_2 \\ \theta^P_3 \end{pmatrix} - \begin{pmatrix} L^{UK}_t \\ S^{UK}_t \\ C^{UK}_t \end{pmatrix} \right] \mathrm{d}t + \begin{pmatrix} \sigma_{11} & 0 & 0 \\ 0 & \sigma_{22} & 0 \\ 0 & 0 & \sigma_{33} \end{pmatrix} \begin{pmatrix} \mathrm{d}W^{1,P}_t \\ \mathrm{d}W^{2,P}_t \\ \mathrm{d}W^{3,P}_t \end{pmatrix}.$$

Likelihood ratio tests of the five parameter restrictions in the preferred $\mathbf{K}^P$ mean-reversion matrix relative to the AFNS model with unrestricted $\mathbf{K}^P$ matrix based on rolling weekly re-estimations since 1995 are shown in Figure 5.[31] The Figure also shows the corresponding likelihood ratio tests for the more parsimonious model with diagonal $\mathbf{K}^P$ matrix favoured by the BIC. The likelihood ratio tests indicate that the restrictions in our preferred AFNS model have been well supported by the data for most of the period, in particular during the 2009–10 period of interest here, while the restrictions in the more parsimonious competitor are typically closer to the border of rejection. Still, for completeness, we consider both the unconstrained AFNS model and the parsimonious independent-factor AFNS model favoured by the BIC.

[30] While the US LSAP programme focused on purchases of debt with maturities of 5–10 years, the UK QE programme purchased a significant amount of debt with a maturity of more than 10 years. However, as in much DTSM analysis, we focus on the yields with a maturity of at most 10 years, in part because these appear to be economically the most relevant ones. The UK data are available at http://www.bankofengland.co.uk/statistics/yieldcurve/index.htm.

[31] Here, we are testing the hypotheses $\kappa^P_{12} = \kappa^P_{13} = \kappa^P_{21} = \kappa^P_{31} = \kappa^P_{32} = 0$ jointly.

Table 9

Evaluation of Alternative Specifications of the AFNS Model of UK Gilt Yields

Alternative Specifications	Goodness-of-fit statistics				
	log L	k	p-value	AIC	BIC
(1) Unrestricted $\mathbf{K}^P$	293,450	24	n.a.	−586,853	−586,690
(2) $\kappa_{13}^P = 0$	293,450	23	1.0000	−586,855	−586,699
(3) $\kappa_{13}^P = \kappa_{32}^P = 0$	293,450	22	0.6547	−586,857	−586,708
(4) $\kappa_{13}^P = \kappa_{32}^P = \kappa_{12}^P = 0$	293,450	21	0.5271	−586,858	−586,716
(5) $\kappa_{13}^P = \ldots = \kappa_{31}^P = 0$	293,450	20	0.3711	−586,860	−586,724
(6) $\kappa_{13}^P = \ldots = \kappa_{21}^P = 0$	293,449	19	0.2733	*−586,860*	−586,731
(7) $\kappa_{13}^P = \ldots = \kappa_{23}^P = 0$	293,446	18	0.0201	−586,857	*−586,735*

Notes. There are seven alternative estimated specifications of the AFNS model of UK gilt yields with the unrestricted 3-by-3 $\mathbf{K}^P$ matrix being the most flexible. Each specification is listed with its maximum log likelihood value (log L), number of parameters (k), the p-value from a likelihood ratio test of the hypothesis that it differs from the specification above with one more free parameter, and the information criteria (AIC and BIC) whose minimum values are shown in italics. The sample is daily from January 2, 1985, to December 31, 2010, a total of 6,535 observations.

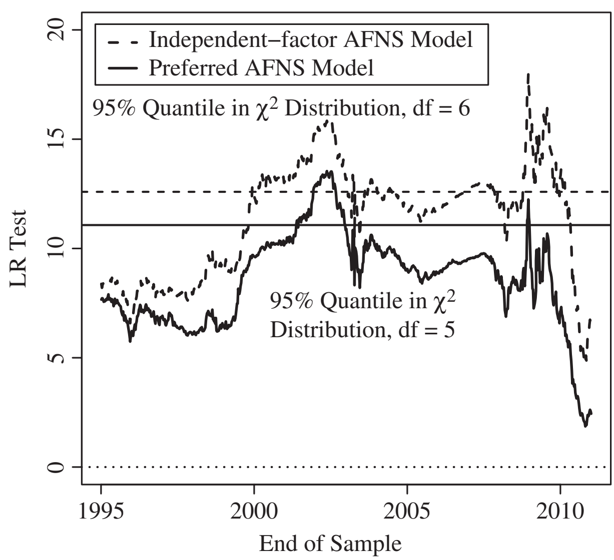

Fig. 5. *Likelihood Ratio Tests of Parameter Restrictions in UK AFNS Models*
Notes. Illustration of the value of likelihood ratio tests of the restrictions imposed in the parsimonious AFNS models relative to the AFNS model with unrestricted $\mathbf{K}^P$ matrix and diagonal $\boldsymbol{\Sigma}$ matrix. The analysis covers weekly re-estimations of expanding samples from January 6, 1995, to December 31, 2010, a total of 835 observations, while the full data set used in the estimation covers the period from January 4, 1985, to December 31, 2010. The 95 percentiles in the relevant χ^2 distributions are shown with horizontal lines.

To quantify the forecast performance of our various AFNS models, Table 10 reports the summary statistics for weekly forecast errors of future Bank Rates one and two years ahead from our empirical AFNS models and from a random walk assumption. Our preferred UK AFNS model specification is comparable to the random walk for this sample.

Table 10

Summary Statistics for Overnight Bank Rate Forecast Errors

Forecasting method	One-year forecast Mean	One-year forecast RMSE	Two-year forecast Mean	Two-year forecast RMSE
Random walk	38.59	137.86	77.36	196.84
Unconstrained AFNS model	−0.71	137.88	56.04	202.41
Indep.-factor AFNS model	52.58	136.05	113.41	202.80
Preferred AFNS model	33.22	131.99	81.41	201.31

Notes. Summary statistics of the forecast errors – mean and root mean-squared errors (RMSEs) – of the overnight target Bank Rate one and two years ahead. The forecasts are weekly starting on January 6, 1995, and running until December 31, 2010, for the one-year forecasts (835 forecasts), and until December 31, 2009, for the two-year forecasts (783 forecasts). All measurements are expressed in basis points.

As in our US analysis, we provide a real-time analysis of bond investors' expectations via a sequence of estimations of the models with expanding samples. Our first estimation sample is from January 2, 1985, to January 2, 1995. Then, we add one additional day of data and re-estimate the model and repeat. Using the estimated models at each date t, we calculate the average expected path for the overnight rate, $(1/\tau)\int_t^{t+\tau} \mathrm{E}_t^P(r_s^{UK})\mathrm{d}s$, as well as the resulting term premium by subtracting that average from the fitted bond yield $\hat{y}_t^{UK}(\tau)$.

3.2. *Response of UK Yields to Bond Purchase Announcements*

Table 11 lists seven key announcements made by the Bank of England's Monetary Policy Committee (MPC) regarding its QE programme.[32] The first six announcement dates are identical to those used by JLST in their analysis of the response of UK gilt yields. The most recent date is the 2011 announcement of further purchases.

We start with a model-free inspection of the response of UK gilt yields on the key QE announcement dates. Cumulated over all events, Table 12 shows that long-term yields declined about 45 basis points on net, while short-term yields fell much less. As such, the one-day reaction in the UK data is smaller than, but qualitatively similar to, the reaction pattern observed in the US data.[33]

To examine whether this response was mirrored in other interest rates, we also report the response of UK OIS rates in Table 13. On net, the long-maturity OIS rates exhibited only about a quarter of the decline registered in comparable gilt yields. This is the main piece of evidence that lead JLST to conclude that the UK QE programme primarily worked through a market segmentation version of the portfolio balance channel. Here, we find further evidence of market segmentation in the response of UK LIBOR and swap interest rates to QE announcements. As shown in Table 14, the

[32] As discussed in the MPC statement following its meeting on March 5, 2009, the MPC views 0.5% as the effective lower boundary for a Bank Rate that is consistent with a sustained smooth operation of related financial markets.

[33] Using a two-day window as in JLST, the reaction in gilt yields with two years or less to maturity remains about the same, while the response of the seven- and 10-year gilts goes above 80 basis points.

Table 11

Key Bank of England QE Announcements

No.	Date	Event	Description
I	February 11, 2009	February Inflation Report	Press conference and inflation report indicated that asset purchases were likely.
II	March 5, 2009	MPC statement	The MPC announced that it would purchase £75 billion of assets over three months. Gilt purchases would be restricted to the 5- to 25-year maturity range.
III	May 7, 2009	MPC statement	The MPC announced that the amount of asset purchases would be extended by a further £ 50 billion to a total of £125 billion.
IV	August 6, 2009	MPC statement	The MPC announced that the amount of asset purchases would be extended to £175 billion and that the buying range would be extended to include gilts with residual maturity greater than three years.
V	November 5, 2009	MPC statement	The MPC announced that the asset purchases would be extended to £200 billion.
VI	February 4, 2010	MPC statement	The MPC announced that the amount of asset purchases would be maintained at £ 200 billion.
VII	October, 6, 2011	MPC statement	The MPC announced that the asset purchases would be extended to £275 billion.

Table 12

Changes in UK Gilt Yields on QE Announcement Dates

	Maturity				
Event	6-month	1-year	2-year	5-year	10-year
I February 11, 2009	−16	−24	−30	−25	−20
II March 5, 2009	0	0	−2	−18	−32
III May 7, 2009	1	0	1	5	6
IV August 6, 2009	1	2	−3	−11	−7
V November 5, 2009	0	0	1	4	7
VI February 4, 2010	0	−1	−2	−2	−1
VII October 6, 2011	1	3	4	3	4
Total net change	−13	−20	−31	−44	−43

Notes. All changes are measured in basis points.

five and 10-year swap interest rates experienced less than half of the declines registered in the comparable gilt yields.[34]

To go beyond this model-free analysis, we use our AFNS models of UK gilt yields to decompose the reaction in the 10-year UK gilt yield into changes in (*i*) a policy expectations component, (*ii*) a term premium component, and (*iii*) a residual component not accounted for by the models. The result of this decomposition on each of the seven QE announcement dates is reported in Table 15. According to the model decompositions, over all seven episodes, policy expectations *did* actually firm between

[34] We were unable to obtain UK corporate bond rate data comparable in quality to the US data, in part because the UK corporate bond market is relatively less liquid.

Table 13

Changes in UK OIS Rates on QE Announcement Dates

Event	Maturity				
	6-month	1-year	2-year	5-year	10-year
I February 11, 2009	−22	−22	−32	−23	−16
II March 5, 2009	11	13	8	−5	−17
III May 7, 2009	0	1	7	14	15
IV August 6, 2009	−2	−8	−8	−2	2
V November 5, 2009	1	0	−5	2	4
VI February 4, 2010	−1	−4	−10	−5	−4
VII October 6, 2011	1	3	8	8	8
Total net change	−12	−18	−32	−12	−8

Notes. All changes are measured in basis points.

Table 14

Changes in UK LIBOR and Swap Rates on QE Announcement Dates

Event	Maturity			
	3-month	2-year	5-year	10-year
I February 11, 2009	−1	−19	−18	−14
II March 5, 2009	−3	−1	−13	−21
III May 7, 2009	0	6	12	13
IV August 6, 2009	0	−8	−4	0
V November 5, 2009	0	−2	1	3
VI February 4, 2010	0	−9	−6	−4
VII October 6, 2011	0	6	7	7
Total net change	−5	−26	−22	−16

Notes. All changes are measured in basis points.

two and twenty basis points depending on the model but this firming was more than offset by declines in term premiums according to all three models.

Given the superior forecast performance of our preferred AFNS model, we focus on that model in the following more detailed analysis. On the first announcement date, the press conference that first indicated that asset purchases by the Bank of England were likely, both policy expectations and term premiums declined by similar magnitudes. However, on the second date, when the first asset purchases were actually announced, there is a clear difference between the reaction of the two components, with policy expectations firming while term premiums declined. Similarly, on August 6, 2009, when the targeted maturity range was extended to encompass gilts with between three and five years remaining to maturity,[35] the difference in the reaction of the two components is equally stark: the firming of policy expectations firming at all horizons was offset by bigger declines in term premiums of approximately the same magnitudes across all maturities. Finally, decomposing the response of the term

[35] Note that gilts with more than 25 years remaining to maturity also became eligible but, because we do not use those maturities in the model estimation, we do not analyse their reaction.

Table 15

Decomposition of Responses of 10-year UK Gilt Yield

Event	Model	Decomposition from models: Avg. target rate next 10 years	10-year term premium	Residual	10-year gilt yield
I February 11, 2009	Unconstrained AFNS	−26	6	−1	−20
	Indep.-factor AFNS	−10	−11	1	
	Preferred AFNS	−12	−9	1	
II March 5, 2009	Unconstrained AFNS	−10	−24	2	−32
	Indep.-factor AFNS	−9	−15	−7	
	Preferred AFNS	17	−41	−7	
III May 7, 2009	Unconstrained AFNS	12	−5	−1	6
	Indep.-factor AFNS	3	2	0	
	Preferred AFNS	−3	9	0	
IV August 6, 2009	Unconstrained AFNS	−4	−4	1	−7
	Indep.-factor AFNS	10	−18	1	
	Preferred AFNS	14	−22	1	
V November 5, 2009	Unconstrained AFNS	5	3	−1	7
	Indep.-factor AFNS	1	6	0	
	Preferred AFNS	−6	13	0	
VI February 4, 2010	Unconstrained AFNS	13	−14	0	−1
	Indep.-factor AFNS	4	−5	0	
	Preferred AFNS	7	−8	0	
VI October 6, 2011	Unconstrained AFNS	15	−12	1	4
	Indep.-factor AFNS	4	0	0	
	Preferred AFNS	4	1	0	
Total net change	Unconstrained AFNS	6	−49	1	−43
	Indep.-factor AFNS	2	−40	−5	
	Preferred AFNS	20	−58	−5	

Notes. The Table contains the decomposition of responses of the 10-year UK gilt yield on seven QE announcement dates into changes in (*i*) the average expected target rate the following 10 years, (*ii*) the 10-year term premium, and (*iii*) the unexplained residual based on empirical DTSMs of UK gilt yields. All changes are measured in basis points.

structure of instantaneous forward rates into forecasted future spot rates and instantaneous forward term premiums leads to similar conclusions. As shown in Figure 6, future forecasted spot rates increased, on net, by about 10–15 basis points at the two to three-year forecast horizon, while the corresponding term premiums declined slightly more than 60 basis points to produce a net decline in the two to three-year forward rates of 50 basis points.

4. Cross-country Yield Responses

In this Section, we examine the reactions of yields in one country to the policy announcements in another to further illuminate the LSAP/QE channels of operation. Announcements of bond purchases in one country could provide investors worldwide with information about the state of the global economy and thus have implications for the outlook for monetary policy in many countries. Alternatively, to the extent that

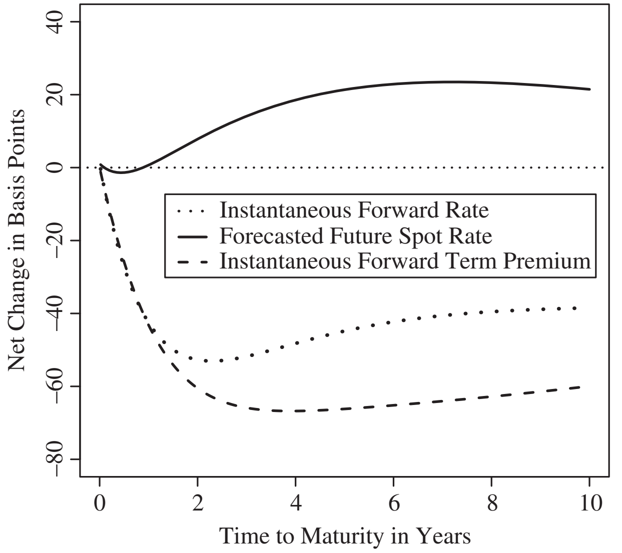

Fig. 6. *Decomposition of Net Response of UK Gilt Forward Rates*
Notes. Illustration of the decomposition of the net response of instantaneous UK gilt forward rates to seven QE announcements into (*i*) forecasted future instantaneous spot rates and (*ii*) instantaneous forward term premiums based on the preferred AFNS model of UK gilt yields.

policy makers generally face similar economic shocks, rely on similar economic models and have similar policy objectives, such announcements could reveal something about the monetary policy reaction function in a variety of countries. For example, in a cross-country signalling channel, the announcement of the US central bank bond purchase could be taken as news about a deepening global economic crisis and lead UK investors to revise down the path for expected future UK policy interest rates. That is, an expectation that the Fed will hold interest rates low for a longer duration could raise the probability of a similar action by the Bank of England. Of course, a portfolio balance channel could operate in the same fashion. Namely, the announcement of US bond purchases could raise the probability of UK bond purchases and the associated expected reduction in UK bond supply could lower UK term premiums.[36]

Thus, the extension of our analysis to consider cross-country effects could potentially provide a useful expansion of our limited sample of LSAP and QE announcement events. In particular, we focus on the response of UK yields to the first four US LSAP announcements that were made before the UK's own QE programme was introduced. These announcements could boost investors' expectations of easier future UK monetary policy, in the form either of a lower expected path for the Bank Rate or of a more likely implementation of a similar UK bond purchase programme. A strong signalling effect from the US policy announcements should affect all UK yields in much the same way as they affected US interest rates for the reasons outlined above.

[36] A direct portfolio balance effect of US purchases on UK term premiums – that is, holding fixed the expected Bank of England balance sheet – seems a remote possibility given the size of global fixed-income markets.

Alternatively, the greater likelihood of a UK bond purchase programme coupled with investors understanding the market segmentation and portfolio balance channel would imply a very modest UK yield response outside of gilts.

As in the previous Sections, we start with a model-free inspection of the observed data. Table 16 reports the response of UK gilt yields to the first four US LSAP announcements. We use a two-day response window because the US announcements occurred after the market close in London. The long-term gilt yields declined slightly more than 60 basis points, while short-term gilt yields declined about 50 basis points. This response is consistent with a signalling spillover effect on UK markets from the US policy actions. Further support for this interpretation is provided in Tables 17 and 18, which report the response of UK OIS, LIBOR and swap rates to the first four US LSAP announcements. These long-term UK rates declined even more than gilt yields, about 70–80 basis points. The peak responses are in the one to five-year contracts, which would naturally decline the most if a prolonged period of low interest rates was expected to be about to start.

Again, to go beyond the observable yield responses, we rely on our empirical DTSMs to decompose the response of the 10-year UK gilt yield to these US LSAP announcements into separate policy expectations and term premium components as well as unexplained residuals. The results of these decompositions are reported in Table 19. On net, all three models agree that declines in policy expectations represented

Table 16

Changes in UK Gilt Yields on US LSAP Announcement Dates Prior to UK QE Programme

	Maturity				
Event	6-month	1-year	2-year	5-year	10-year
I November 25, 2008	−8	4	14	−1	−16
II December 1, 2008	−26	−35	−42	−33	−23
III December 16, 2008	−15	−20	−24	−24	−24
IV January 28, 2009	1	0	−3	−4	2
Total net change	−49	−51	−56	−62	−61

Notes. All changes are measured in basis points.

Table 17

Changes in UK OIS Rates on US LSAP Announcement Dates Prior to UK QE Programme

	Maturity				
Event	6-month	1-year	2-year	5-year	10-year
I November 25, 2008	−6	−14	−17	−15	−15
II December 1, 2008	−32	−30	−32	−32	−30
III December 16, 2008	−33	−31	−27	−19	−18
IV January 28, 2009	0	−4	−6	−6	−5
Total net change	−71	−80	−81	−72	−68

Notes. All changes are measured in basis points.

Table 18

Changes in UK LIBOR and Swap Rates on US LSAP Announcement Dates Prior to UK QE Programme

	Maturity			
Event	3-month	2-year	5-year	10-year
I November 25, 2008	−4	−14	−15	−16
II December 1, 2008	−7	−38	−31	−29
III December 16, 2008	−8	−23	−23	−17
IV January 28, 2009	0	−8	−6	−2
Total net change	−19	−74	−75	−64

Notes. All changes are measured in basis points.

Table 19

Decomposition of Responses of 10-year UK Gilt Yield on US LSAP Announcement Dates Prior to UK QE Programme

		Decomposition from models			
Event	Model	Avg. target rate next 10 years	10-year term premium	Residual	10-year gilt yield
I November 25, 2008	Unconstrained AFNS	3	−25	6	−16
	Indep.-factor AFNS	−32	15	1	
	Preferred AFNS	6	−23	1	
II December 1, 2008	Unconstrained AFNS	−40	18	−2	−23
	Indep.-factor AFNS	−8	−13	−3	
	Preferred AFNS	−30	10	−3	
III December 16, 2008	Unconstrained AFNS	−25	1	0	−24
	Indep.-factor AFNS	−15	−8	−1	
	Preferred AFNS	−19	−4	−1	
IV January 28, 2009	Unconstrained AFNS	3	0	0	2
	Indep.-factor AFNS	5	−5	3	
	Preferred AFNS	−1	0	3	
Total net change	Unconstrained AFNS	−59	−5	4	−61
	Indep.-factor AFNS	−50	−11	1	
	Preferred AFNS	−44	−18	1	

Notes. The Table contains the decomposition of two-day responses of the 10-year UK gilt yield on the four US LSAP announcement dates that occurred prior to the introduction of the UK QE programme. The gilt yield changes are decomposed into (*i*) the average expected target rate the following 10 years, (*ii*) the 10-year term premium, and (*iii*) the unexplained residual based on empirical DTSMs of UK gilt yields. All changes are measured in basis points.

two-thirds or more of the declines observed in the 10-year gilt yield in response to these announcements. This suggests the presence of a strong signalling channel that affected not just US yields, but also overseas markets.[37]

[37] Neely (2012) also reports strong cross-country government bond yield responses in Australia, Canada, Germany, Japan and the UK to these US announcements. Furthermore, he presents intraday data on government bond futures prices and foreign exchange rates that indicate that the market response was complete within a few hours for the five first US LSAP announcements that he analyses, which suggests that the overseas effects were not likely reflecting other news.

To summarise, there is essentially no evidence of a portfolio balance or market segmentation channel in the UK response to these US LSAP announcements, even though UK market stress was likely more intense than later on when the UK QE programme was announced. That is, the news of Fed bond purchases seemed to signal a longer period of low UK short-term interest rates rather than a future programme of UK QE and reduced UK bond supply. Hence, we see no reason why a market-wide signalling effect could not have occurred in response to the UK QE announcements, if investors had interpreted it that way.

For symmetry, we also examined the response of US interest rates to the UK QE announcements. Perhaps not too surprisingly, Treasury and swap market yields and our model decompositions of the 10-year US Treasury yield generally indicated little reaction to the UK QE announcements and only modest declines in US term premiums.[38] This is consistent with our earlier results that the response to the UK QE announcements was concentrated in the gilt market.

5. Conclusion

The existing literature on the response of fixed-income markets to the Federal Reserve's first LSAP programme and the Bank of England's QE programme suggests a negative effect of between 50 and 100 basis points on 10-year yields. To understand these results, we used empirical DTSMs for each country to decompose the yield responses to key announcements regarding the bond purchase programme. For the US, our results suggest that a key effect of the Fed's LSAP programme was to lower policy expectations. In contrast, for the UK, yield declines following QE announcements appear to have been entirely driven by reductions in term premiums. Of course, as noted above and stressed in Bauer and Rudebusch (2011), the uncertainty regarding these conclusions is not negligible.

The differences between the US and UK reactions of the expectations and term premium components of longer term yields to central bank bond purchases are notable – especially given the similar bond purchase amounts and underlying macroeconomic rationales in the two countries. The contrasting channels of influence of the US and UK unconventional policy can perhaps be traced to differences in policy communication and financial market structure. Specifically, with regard to communication, the Federal Reserve was clearly more willing to provide monetary policy forward guidance near the zero bound. For example, the FOMC statement released following its December 16, 2008, meeting noted that 'the Committee anticipates that weak economic conditions are likely to warrant exceptionally low levels of the federal funds rate for some time.'[39] The FOMC announcements of bond purchases could have been interpreted as reinforcing this guidance and essentially providing a signal that the period of low funds rate levels was even longer. In contrast, forward-looking policy guidance on interest rates was absent from the UK MPC statements and the signalling

[38] US yields did experience sizeable movements on three dates but that variation appears driven by non-UK-related news. Complete results are available from the authors.

[39] At the March 18, 2009, meeting, this timing language was modified to 'for an extended period', and later, specific dates were provided.

value of the QE programme may have been commensurately diminished.[40] A separate reason for the disparate role of the term premium in the UK and US reactions of longer term yields to bond purchase programmes could perhaps be the differences in financial market structures across the two countries. For the operation of a portfolio balance channel, the exact nature of investors' preferred habitats and limits on arbitrage crucially determine the magnitude of shifts in term premiums. US government bond markets are widely considered more liquid than UK markets and US Treasury securities are held by a broader class of international investors. Therefore, institutional and investor differences in financial markets may also play a role in explaining the different reactions across countries.

Appendix: AFNS Model Estimation Methodology

We estimate the AFNS models by maximising the likelihood function in the standard Kalman filter algorithm, which is an efficient and consistent estimator in this affine Gaussian setting (Harvey, 1989). In the continuous-time formulation of the AFNS model, the conditional mean vector and the conditional covariance matrix are given by

$$\mathrm{E}^P(\mathbf{x}_T|\mathcal{F}_t) = [\mathbf{I} - \exp(-\mathbf{K}^P\Delta t)]\boldsymbol{\theta}^P + \exp(-\mathbf{K}^P\Delta t)\mathbf{x}_t,$$
$$V^P(\mathbf{x}_T|\mathcal{F}_t) = \int_0^{\Delta t} \mathrm{e}^{-\mathbf{K}^P s}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathrm{e}^{-(\mathbf{K}^P)' s}\mathrm{d}s,$$

where $\Delta t = T - t$.

The state equation, which represents the factor dynamics under the P-measure, is given by

$$\mathbf{x}_t = [\mathbf{I} - \exp(-\mathbf{K}^P\Delta t)]\boldsymbol{\theta}^P + \exp(-\mathbf{K}^P\Delta t)\mathbf{x}_{t-1} + \eta_t,$$

where Δt is the time between observations. The conditional covariance matrix for the shock terms is given by[41]

$$\mathbf{Q} = \int_0^{\Delta t} \mathrm{e}^{-\mathbf{K}^P s}\boldsymbol{\Sigma}\boldsymbol{\Sigma}'\mathrm{e}^{-(\mathbf{K}^P)' s}\mathrm{d}s.$$

The AFNS measurement equation is given by

$$y_t(\tau) = \tilde{a}(\tau) + \tilde{\mathbf{b}}(\tau)'\mathbf{x}_t + \varepsilon_t(\tau),$$

where $\tilde{a}(\tau) = -\frac{1}{\tau}a(\tau)$ and $\tilde{\mathbf{b}}(\tau) = -\frac{1}{\tau}\mathbf{b}(\tau)$ are as described in (2).

The error structure is assumed to be

$$\begin{pmatrix} \boldsymbol{\eta}_t \\ \boldsymbol{\varepsilon}_t \end{pmatrix} \sim \mathrm{N}\left[\begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}, \begin{pmatrix} \mathbf{Q} & \mathbf{0} \\ \mathbf{0} & \mathbf{H} \end{pmatrix}\right],$$

where $\mathbf{H}$ is a diagonal matrix

$$\mathbf{H} = \begin{pmatrix} \sigma^2(\tau_1) & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & \sigma^2(\tau_N) \end{pmatrix}.$$

[40] The operation of the UK QE programme was also implemented with less forward guidance, with each step of the programme intended to be completed in three months or less. In contrast, the US LSAP programme involved longer periods of purchases, on the order of nine months.

[41] In the estimation, we calculate the conditional and unconditional covariance matrices using the analytical solutions provided in Fisher and Gilles (1996).

The linear least-squares optimality of the Kalman filter requires that the transition and measurement errors be orthogonal to the initial state, i.e.

$$\mathrm{E}^P(\mathbf{x}_0\boldsymbol{\eta}_t') = \mathbf{0}, \quad \mathrm{E}^P(\mathbf{x}_0\boldsymbol{\varepsilon}_t') = \mathbf{0}.$$

Finally, parameter standard deviations are calculated as

$$\boldsymbol{\Sigma}(\hat{\boldsymbol{\psi}}) = \frac{1}{T}\left[\frac{1}{T}\sum_{t=1}^{T}\frac{\partial \log l_t(\hat{\boldsymbol{\psi}})}{\partial \boldsymbol{\psi}}\frac{\partial \log l_t(\hat{\boldsymbol{\psi}})'}{\partial \boldsymbol{\psi}}\right]^{-1},$$

where $\hat{\boldsymbol{\psi}}$ denotes the estimated model parameter set.

Normally, we start the Kalman filter using the unconditional distribution of the state variables. However, when we impose a unit-root property on the Nelson–Siegel level factor, the joint dynamics of the state variables are no longer stationary. By implication, we cannot start the Kalman filter at the unconditional distribution. Instead, we follow Duffee (1999) and derive a distribution for the starting point of the Kalman filter based on the yields observed at the first data point in each sample. Specifically, the model states that zero-coupon yields are given by

$$\mathbf{y}_t = \tilde{\mathbf{a}} + \tilde{\mathbf{B}}\mathbf{x}_t + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim \mathrm{N}(\mathbf{0}, \mathbf{H}).$$

For the first set of observations, this equation reads

$$\mathbf{y}_1 = \tilde{\mathbf{a}} + \tilde{\mathbf{B}}\tilde{\mathbf{x}}_0 + \tilde{\boldsymbol{\varepsilon}}_0 \Longleftrightarrow \tilde{\mathbf{B}}\tilde{\mathbf{x}}_0 = \mathbf{y}_1 - \tilde{\mathbf{a}} - \tilde{\boldsymbol{\varepsilon}}_0.$$

Now, multiply from the left on both sides by $\tilde{\mathbf{B}}'$ to obtain

$$\tilde{\mathbf{B}}'\tilde{\mathbf{B}}\tilde{\mathbf{x}}_0 = \tilde{\mathbf{B}}'(\mathbf{y}_1 - \tilde{\mathbf{a}}) - \tilde{\mathbf{B}}'\tilde{\boldsymbol{\varepsilon}}_0.$$

We can then isolate $\tilde{\mathbf{x}}_0$ by using the inverse of $\tilde{\mathbf{B}}'\tilde{\mathbf{B}}$

$$\tilde{\mathbf{x}}_0 = (\tilde{\mathbf{B}}'\tilde{\mathbf{B}})^{-1}\tilde{\mathbf{B}}'(\mathbf{y}_1 - \tilde{\mathbf{a}}) - (\tilde{\mathbf{B}}'\tilde{\mathbf{B}})^{-1}\tilde{\mathbf{B}}'\tilde{\boldsymbol{\varepsilon}}_0.$$

Here, $\tilde{\boldsymbol{\varepsilon}}_0$ is normally distributed with a mean of zero and a variance matrix equal to $\mathbf{H}$. By implication, $\tilde{\mathbf{x}}_0$ follows a normal distribution with the following properties

$$\tilde{\mathbf{x}}_0 \sim N[(\tilde{\mathbf{B}}'\tilde{\mathbf{B}})^{-1}\tilde{\mathbf{B}}'(\mathbf{y}_1 - \tilde{\mathbf{a}}), (\tilde{\mathbf{B}}'\tilde{\mathbf{B}})^{-1}\tilde{\mathbf{B}}'\mathbf{H}\tilde{\mathbf{B}}(\tilde{\mathbf{B}}'\tilde{\mathbf{B}})^{-1}].$$

Thus, this is the normal distribution used to start the Kalman filter when unit-root properties are imposed.[42]

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