A MACRO-FINANCE MODEL OF THE TERM STRUCTURE, MONETARY POLICY AND THE ECONOMY*

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This article develops and estimates a macro-finance model that combines a canonical affine no-arbitrage finance specification of the term structure of interest rates with standard macroeconomic aggregate relationships for output and inflation. Based on this combination of yield curve and macroeconomic structure and data, we obtain several interesting results: (1) the latent term structure factors from no-arbitrage finance models appear to have important macroeconomic and monetary policy underpinnings, (2) there is no evidence of a slow partial adjustment of the policy interest rate by the central bank, and (3) both forward-looking and backward-looking elements play roles in macroeconomic dynamics.

Bonds of various maturities all trade simultaneously in a well-organised market at prices that appear to preclude residual opportunities for riskless financial arbitrage. Indeed, the assumption of no arbitrage is central to an enormous finance literature that is devoted to the empirical analysis of bond pricing and the yield curve. This research has found that almost all movements in the yield curve can be captured in a no-arbitrage framework in which yields are linear functions of a few unobservable or latent factors (Duffie and Kan, 1996; Dai and Singleton, 2000). However, while these popular affine no-arbitrage models do provide useful statistical descriptions of the term structure, they offer little insight into the economic nature of the underlying latent factors or forces that drive movements in interest rates. To provide such insight, this article combines a canonical affine no-arbitrage model of the term structure with a fairly standard macroeconomic representation of output and inflation.

The finance and macroeconomic research literatures have developed, to a remarkable extent, in isolation of each other, despite sharing many objects of keen mutual interest, notably, the short-term interest rate. From a finance perspective, the short rate is a fundamental building block for rates of other maturities because long yields are risk-adjusted averages of expected future short rates. From a macro perspective, the short rate is a key policy instrument under the direct control of the central bank, which adjusts the rate in order to achieve the economic stabilisation goals of monetary policy. Taken together, a joint macro-finance perspective would suggest that understanding the manner in which central banks move the short rate in response to fundamental macroeconomic shocks should explain movements in the short end of the yield curve; furthermore, with the consistency between long and short rates enforced by the finance no-arbitrage assumption, expected future macroeconomic variation should account for movements farther out on the yield curve as well.

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This article provides an example of a model that takes such a joint perspective, which we find yields interesting synergetic results. For example, in our combined macro-finance analysis, we find that the standard no-arbitrage term structure factors do have clear macroeconomic underpinnings. Therefore, an explicit macro structure can help to provide insight into the behaviour of the yield curve beyond what a pure finance model can suggest. Conversely, a joint macro-finance perspective can also illuminate various macroeconomic issues, since the addition of term structure information to a macroeconomic model can help to sharpen inference. Specifically, the term structure factors summarise expectations about future short rates, which in turn reflect expectations about the future dynamics of the economy, and with forward-looking economic agents, these expectations should be important determinants of current and future macroeconomic variables. The relative importance of forward versus backward-looking elements in the dynamics of the economy is an important unresolved issue in macroeconomics that the incorporation of term structure information may help to resolve. Also, the inclusion of term structure information can shed light on the issue of whether central banks engage in interest rate smoothing or gradual partial adjustment in setting monetary policy, as in Rudebusch (2002b, 2006).

We begin our analysis in Section 1 by estimating an off-the-shelf affine no-arbitrage model of the yield curve using data on yields but not macroeconomic variables. This standard ‘yields-only’ finance model introduces the affine, no-arbitrage structure and provides a useful benchmark. The yields-only model expresses the short rate as the sum of various latent factors that have no economic content. In contrast, in Section 2, we introduce our ‘macro-finance’ model that adds macroeconomic content and relates the short rate directly to macroeconomic fundamentals through a monetary policy reaction function (in the manner of the popular Taylor rule, in which the short rate depends on inflation and output). This representation combines an affine no-arbitrage term structure with a small macroeconomic model that has rational expectations as well as inertial elements. The unified macro-finance model is estimated from the data by maximum likelihood methods and demonstrates a fit and dynamics comparable to the separate yields-only model and a stand-alone macroeconomic model. This new framework is able to interpret the latent factors of the yield curve in terms of macroeconomic variables, with one of the latent factors identified as a perceived inflation target and the other as a cyclical monetary policy response to the economy. It also sheds light on the importance of inflation and output expectations in the economy and the extent of monetary policy inertia or partial adjustment. Section 3 concludes with an overview of the strengths and weaknesses of a combined macro-finance approach.

The broad contour of our analysis is consistent with much recent research that relates the general level of interest rates to an expected underlying inflation component and the slope or tilt of the yield curve to monetary policy actions. However, there are three distinctive features of our macro-finance analysis. First, we use a structural macroeconomic specification, as in Rudebusch (2002a) and Hördahl et al. (2006). Other papers have related macro variables to the yield curve using little or no macroeconomic structure, including, for example, Ang and Piazzesi (2003), Ang et al. (2006), Dewachter and Lyrio (2006), Kozicki and Tinsley (2001) and Diebold et al. (2006). Second, following the vast finance literature, we use an affine no-arbitrage structure in

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which the yield curve (and the price of risk) depends on a few latent factors. This arrangement allows a clear comparison of the term structure elements in our model to parallel one in the existing finance literature. Finally, as in Diebold et al. (2006), our model also allows for a bidirectional feedback between the term structure factors and macro variables. In contrast, as in Ang and Piazzesi (2003), the macro sector is often modelled as completely exogenous to the yield curve.

1. A No-Arbitrage Yields-Only Model

The canonical finance term structure model contains three basic equations. The first is the transition equation for the state vector relevant for pricing bonds. We assume there are two latent factors $L_t$ and $S_t$ and that the state vector, $F_t = (L_t, S_t)'$, is a Gaussian VAR(1) process:

$$F_t = \rho F_{t-1} + \Sigma \varepsilon_t, \quad (1)$$

where $\varepsilon_t$ is i.i.d. $N(0, I_2)$, $\Sigma$ is diagonal, and $\rho$ is a 2×2 lower triangular matrix. The second equation defines the one-period short rate $i_t$ to be a linear function of the latent variables with a constant $\delta_0$:

$$i_t = \delta_0 + L_t + S_t = \delta_0 + \delta_1 F_t. \quad (2)$$

Without loss of generality, (2) implies unitary loadings of the two factors on the short rate because of the normalisation of these unobservable factors. Finally, the price of risk associated with the shocks $\varepsilon_t$ is defined to be a linear function of the state of the economy:

$$K_t = K_L K_S = k_0 + k_1 F_t. \quad (3)$$

The state transition equation (1), the short rate equation (2) and the price of risk (3) form a discrete-time Gaussian two-factor term structure model. In such a structure, the logarithm of the price of a $j$-period zero-coupon bond is a linear function of the factors

$$\ln(b_{j,t}) = A_j + B_j F_t, \quad (4)$$

where (Ang and Piazzesi, 2003) the coefficients $A_j$ and $B_j$ are recursively defined by

$$A_1 = -\delta_0; \quad B_1 = -\delta_1 \quad (5)$$

$$A_{j+1} - A_j = B_j (-\Sigma \lambda_0) + \frac{1}{2} B_j \Sigma \Sigma B_j + A_1 \quad (6)$$

$$B_{j+1} = B_j (\rho - \Sigma \lambda_1) + B_1; \quad j = 1, 2, \ldots, J. \quad (7)$$

Given this bond pricing, the continuously compounded yield to maturity $i_{j,t}$ of a $j$-period nominal zero-coupon bond is given by the linear function

$$i_{j,t} = -\ln(b_{j,t})/j = A_j + B_j F_t, \quad (8)$$

where $A_j = -A_j/j$ and $B_j = -B_j/j$. 

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For a given set of observed yields, the likelihood function of this model can be calculated in closed form and the model can be estimated by maximum likelihood. We estimate this model using end-of-month data from January 1988 to December 2000 on five US Treasury zero-coupon yields that have maturities of 1, 3, 12, 36, and 60 months. (The yields are unsmoothed Fama-Bliss data expressed at an annual rate in percentages and kindly provided by Robert Bliss.) Since there are two underlying latent factors but five observable yields, we follow the usual strategy and assume that the 3, 12, and 36-month yields are measured with i.i.d. error, as in Ang and Piazzesi (2003). The estimated size of such measurement error is one common metric to assess model fit.

We limit the estimation sample in order to increase the chance that it is drawn from a single stable period of monetary policy behaviour. Over the entire post-war sample, the reaction of the Federal Reserve in adjusting the short rate in response to macroeconomic shocks appears to have changed. In particular, the Fed’s short rate response to changes in inflation during the 1970s has been found to be less vigorous than in the 1990s. In addition, Rudebusch and Wu (2007) find significant evidence of a change in the pricing of risk since the mid-1980s. Such changes alter the relationship between the term structure and macroeconomic variables. To avoid such instability, our fairly short sample period falls completely within Alan Greenspan’s tenure as Fed Chairman, which is often treated as a consistent monetary policy regime.

For our sample, just two factors appear sufficient to capture movements in the yield curve. This perhaps reflects the exclusion from our sample of the period of heightened interest rate volatility during the late 1970s and early 1980s. One indication of the superfluous nature of a third factor is provided by a principal component analysis. In our sample, the first two principal components capture 93.3% of the variation in the five yields and so can account for essentially all of the movements in the yield curve.

The parameter estimates of the yields-only model are reported in Table 1.2 As is typically found in empirical estimates of such a term structure model, the latent factors differ somewhat in their time-series properties as shown by the estimated $\rho$. The factor $L_t$ is very persistent, while $S_t$ is less so. The parameters in $\lambda_1$, which determine the time variation in the price of risk, appear significant and the model fits the 3, 12 and 36-month rates with fairly typical measurement error standard deviations of 20, 35 and 16 basis points, respectively.3

Some insight into the workings of the yields-only model is provided in Figure 1, which shows the initial response of yields of various maturities to a one percentage point increase in each factor – the ‘factor loadings’. A positive shock to $L_t$ raises the

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1 For example, see Fuhrer (1996), Judd and Rudebusch (1998), Clarida et al. (2000) and Rudebusch (2005).
2 Note that in the pricing formula (6), the constant $\lambda_0$ only enters the definition of $\bar{A}_t$, so changes in $\lambda_0$ affect the steady-state shape of the yield curve and not its variation over time. To reduce the number of parameters to be estimated, we impose the restriction that $\lambda_0 = 0$. Accordingly, we de-mean the bond yields and focus on the variations of yields from sample averages in the model estimation. This procedure may slightly alter the parameter estimates since the effects of $\bar{B}$ and $\Sigma$ on $\bar{A}_t$, through a Jensen's inequality term will be ignored. However, in practice, the amount of information contained in this term is trivial, especially in the present model estimation where the longest maturity is 5 years.
3 For comparison, the pricing errors in the three-factor yields-only Ang and Piazzesi (2003) model are 25 and 11 basis points for the 3 and 36-month yields (properly annualised).
yields of all maturities by almost an identical amount. This effect induces an essentially parallel shift in the term structure that boosts the level of the whole yield curve, so the \(L_t\) factor is often called a ‘level’ factor, which is a term we will adopt. Likewise, a positive shock to \(S_t\) increases short-term yields by much more than the long-term yields, so the yield curve tilts and becomes less steeply upward sloped (or more steeply downward sloped); thus, this factor is termed the ‘slope’ factor. Overall, these estimates and loadings reveal an empirical no-arbitrage model – even for our short sample – that is quite consistent with existing estimated models in the empirical finance literature on bond pricing.

Table 1

<table>
<thead>
<tr>
<th>Factor dynamics ((\rho))</th>
<th>(L_{t-1})</th>
<th>(S_{t-1})</th>
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</thead>
<tbody>
<tr>
<td>(L_t)</td>
<td>0.997</td>
<td>—</td>
</tr>
<tr>
<td>(S_t)</td>
<td>0.021 (0.0013)</td>
<td>0.945 (0.0039)</td>
</tr>
</tbody>
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<tr>
<th>Risk price ((\lambda))</th>
<th>(L_t)</th>
<th>(S_t)</th>
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</thead>
<tbody>
<tr>
<td>(\lambda_{L, t})</td>
<td>-0.0148 (0.0013)</td>
<td>0.0032 (0.0014)</td>
</tr>
<tr>
<td>(\lambda_{S, t})</td>
<td>-0.0028 (0.0014)</td>
<td>-0.0095 (0.0014)</td>
</tr>
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<tr>
<th>Standard deviations ((\Sigma))</th>
<th>(L_t)</th>
<th>(S_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_L)</td>
<td>0.271</td>
<td>(0.0098)</td>
</tr>
<tr>
<td>(\sigma_S)</td>
<td>0.443</td>
<td>(0.0077)</td>
</tr>
</tbody>
</table>

<table>
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<tr>
<th>Standard deviations of measurement error</th>
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<tbody>
<tr>
<td>3-month</td>
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<tr>
<td>12-month</td>
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<tr>
<td>36-month</td>
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*Note.* Standard errors of the estimates are in parentheses.

![Fig. 1. Factor Loadings of Yields-Only Model](image)

*Note:* These factor loadings show the impact response from a 1 percentage point increase in level or slope on the yield of a given maturity.
2. A Macro-Finance Model

The above finance model decomposes the short-term interest rate into unobserved factors that are modelled as autoregressive time series unrelated to macroeconomic variation. In contrast, from a macro perspective, the short rate is determined by macroeconomic variables in the context of a monetary policy reaction function. This Section tries to reconcile these two views by presenting a combined macro-finance model in which the term structure factors are jointly estimated with macroeconomic relationships.

2.1. Model Structure

In the macro-finance model, the one-month short rate is defined to be the sum of two latent term structure factors

\[ i_t = \delta_0 + L^m_t + S^m_t, \]  
\[ \text{(9)} \]

as in a typical affine no-arbitrage term structure representation, where \( L^m_t \) and \( S^m_t \) denote unobserved macro-finance term structure factors. Of course, the estimated macro-finance factors, \( L^m_t \) and \( S^m_t \), may differ from their yields-only counterparts \( L_t \) and \( S_t \); however, as shown below, differences between the realised historical time series of these factors are very small (although there are important differences in the dynamic impulse responses).

In terms of structure, the dynamics of the macro-finance latent factors are specified as

\[ L^m_t = \rho_L L^m_{t-1} + (1 - \rho_L) \pi_t + \varepsilon_{L,t}, \]  
\[ \text{(10)} \]

\[ S^m_t = \rho_S S^m_{t-1} + (1 - \rho_S) [g y_t + g_e (\pi_t - L^m_t)] + u_{S,t}, \]  
\[ \text{(11)} \]

\[ u_{S,t} = \rho_u u_{S,t-1} + \varepsilon_{S,t}, \]  
\[ \text{(12)} \]

where \( \pi_t \) is the annual inflation rate and \( y_t \) is a measure of the output gap.\(^4\)

These equations provide macroeconomic underpinnings for the latent term structure factors. In (10), the factor \( L^m_t \) is interpreted as the underlying rate of inflation, essentially the central bank’s implicit inflation target as perceived by private agents. The general identification of the overall level of interest rates with the perceived inflation goal of the central bank is a common theme in the recent macro-finance literature (notably, Kozicki and Tinsley, 2001; Dewachter and Lyrio, 2006; and Hördacl et al., 2006). Indeed, based on the close association between the level factor and inflation expectations obtained from readings on indexed debt, Barr and Campbell (1997) conclude that in the UK ‘almost 80 percent of the movement in long-term nominal rates appears to be due to changes in expected long-term inflation’. In the US, Gürkaynak et al. (2005) also argue that movements in long rates reflect fluctuations in inflation perceptions and not real rates. Therefore, as a first approximation, we assume

\[ \text{For the empirical analysis with monthly data, } \pi_t \text{ is defined as the 12-month percentage change in the personal consumption expenditures price index (} P_t \text{) in percent at an annual rate (i.e., } \pi_t \equiv 12(p_t - p_{t-1}), \text{ where } p_t = 100 \ln P_t, \text{ and } y_t \text{ is de-meaned industrial capacity utilisation.} \]

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that month-to-month variation in medium-term inflation expectations dominate level factor fluctuations during our 1988 to 2000 sample; furthermore, agents are assumed to slowly modify their views about \( L^m_t \) as actual inflation changes. (As we shall see from the empirical factor loadings below, \( L^m_t \) will be associated with the level of yields with maturities from 2 to 5 years, which is an important indication of the appropriate horizon to associate with the inflation expectations embodied in \( L^m_t \).)

In (11), which mimics the classic Taylor rule, the slope factor \( S^m_t \) captures the central bank’s dual mandate to stabilise the real economy and keep inflation close to its target level. Given the 2 to 5-year horizon of inflation expectations embodied in \( L^m_t \), we believe this factor represents an interim or medium-term inflation target, as in Bomfim and Rudebusch (2000). Accordingly, in (11), the central bank is assumed to be attempting to close the gap between actual inflation and this interim inflation target. In addition, the dynamics of \( S^m_t \) allow for both partial adjustment and serially correlated shocks. The first type of dynamic behaviour is often called monetary policy inertia or interest rate smoothing, and it has been used by Clarida et al. (2000) and many others. In contrast, a second interpretation of the dynamics of \( S^m_t \) is that the \( u_S,t \) are the effect of policy responses to special circumstances and information that were not captured by the simple Taylor rule specification but were important to policy makers, as described in Rudebusch (2002b, 2006).  

Choosing between the partial adjustment and serially correlated shocks specifications depends crucially on separating the influences of contemporaneous and lagged regressors in the reaction function, which are typically difficult to untangle in a single equation context. As Rudebusch (2002b) stresses, this problem is particularly acute for estimated monetary policy rules, because uncertainty in modelling the desired policy rate (given the endogeneity of regressors, the real-time nature of the information set, and the small samples available) makes the single-equation evidence on the rule’s dynamic specification suspect. Thus, a policy rule with slow partial adjustment and no serial correlation in the errors will be difficult to distinguish empirically from a policy rule that has immediate policy adjustment but highly serially correlated shocks. However, information contained in the term structure can help to distinguish between these two interpretations. Our macro-finance model will allow for both types of policy rule dynamics. Specifically, if \( \rho_u = 0 \), the dynamics of \( S^m_t \) arise from monetary policy partial adjustment. Conversely, if \( \rho_S = 0 \), the dynamics reflect the Fed’s reaction to serially correlated information or events not captured by output and inflation.

We close the above equations with a standard small macroeconomic model of inflation and output that can flexibly model the relative contribution of explicitly forward-looking versus backward-looking behaviour in the determination of the macroeconomic variables. Much of the appeal of this so-called New Keynesian

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5 These rule deviations are not exogenous monetary policy shocks that represent actions independent of the economy; instead, they are endogenous responses to a variety of influences that cannot be captured by some easily observable variable such as output or inflation. For example, persistent deviations between the true and perceived inflation goals could show up as serially correlated residuals.

6 In particular, Rudebusch (2002b, 2006) demonstrate that a slow partial adjustment of the short rate to new information by the Fed should imply the existence of predictable future variation in the short rate that is not present with serially correlated shocks. In fact, the general lack of predictive information in the yield curve about changes in the short rate suggests the absence of policy inertia.
specification is its theoretical foundation in a dynamic general equilibrium theory with temporary nominal rigidities; however, we focus on just the two key aggregate relationships for output and inflation. For inflation, a standard theoretical representation is given by

$$\pi_t = \mu_\pi E_t \pi_{t+1} + (1 - \mu_\pi)\pi_{t-1} + \alpha_y y_t + \varepsilon_{\pi,t},$$

where $E_t \pi_{t+1}$ is the expectation of period $t + 1$ inflation conditional on a time $t$ information set.\(^7\) In this specification, the current (one-period) inflation rate is determined by rational expectations of future inflation, lagged inflation and output. A key parameter is $\mu_\pi$, which measures the relative importance of forward versus backward-looking pricing behaviour.\(^8\) Since our model is estimated with monthly data, its empirical specification differs from (13). Given the institutional length of price contracts in the real world, the one-period leads and lags in theory are typically assumed to pertain to periods much longer than one month; indeed, empirical macroeconomic analyses invariably use data sampled at a quarterly or even annual frequency. For estimation with monthly data, we reformulate (13), with longer leads and lags as,

$$\pi_t = \mu_\pi L^m_t + (1 - \mu_\pi)(\alpha_{\pi_1} \pi_{t-1} + \alpha_{\pi_2} \pi_{t-2}) + \alpha_y y_t + \varepsilon_{\pi,t}. \tag{14}$$

In this specification, inflation in the current month is set as a weighted average of the public’s expectation of the medium-term inflation target, which we identify as $L^m_t$, and two lags of inflation. Also, there is a one-month lag on the output gap to reflect the usual adjustment costs and recognition lags.

The standard New Keynesian theory of aggregate demand can be represented by an intertemporal Euler equation of the form:

$$y_t = \mu_y E_t y_{t+1} + (1 - \mu_y) y_{t-1} - \beta_r(i_t - E_t \pi_{t+1}) + \varepsilon_{y,t}. \tag{15}$$

Current output is determined by expected future output, $E_t y_{t+1}$, lagged output and the ex ante real interest rate. The parameter $\mu_y$ measures the relative importance of expected future output versus lagged output, where the latter term is crucial to account for real-world costs of adjustment and habit formation (Fuhrer and Rudebusch, 2004). For empirical implementation with monthly data, we estimate an equation of the form:

$$y_t = \mu_y E_t y_{t+1} + (1 - \mu_y)(\beta_{y1} y_{t-1} + \beta_{y2} y_{t-2}) - \beta_r(i_t - E_t \pi_{t+1}) + \varepsilon_{y,t}. \tag{16}$$

This equation has an additional lag of output but the key difference is the specification of the ex ante real interest rate, which is proxied by $i_t - L^m_{t-1}$; that is, agents judge nominal rates against their view of the underlying future inflation not just next month’s inflation rate. Also, because our interest rate data are end-of-month observations, the $t - 1$ timing of the real rate is appropriate for the determination of time $t$ output.

\(^7\) As above, data are de-meaned, so no constants are included in the macro equations.

\(^8\) As a theoretical matter, the value of $\mu_\pi$ is not clearly determined. From well-known models of price-setting behaviour, it is possible to derive an inflation equation with $\mu_\pi \approx 1$. However, many authors assume that with realistic costs of adjustment and overlapping price and wage contracts there will be some inertia in inflation, so $\mu_\pi$ will be less than one (Svensson 1999; Fuhrer and Moore, 1995).
It is useful to note that although the only nominal rate to explicitly enter the aggregate demand equation is the current short rate, by iterating this equation forward, it can be re-written in a form where \( y_t \) depends on the sum of expected future short rates (Fuhrer and Rudebusch, 2004). Accordingly, there is some link between long nominal rates and aggregate demand through the path of future short rates. Still, in this aggregate demand specification, shifts in the risk premium component of long rates do not affect output directly, which is also true in a wide variety of standard theoretical models (McGough et al., 2005; Rudebusch et al., 2007). It may be interesting to include longer-maturity interest rates (with embedded term premiums) as a direct determinant of output but this is computationally difficult because the implied model must be solved numerically instead of applying the usual linear solution algorithms.

The factor \( L^m_t \), which we interpret as medium-term inflation expectations, enters the macro-finance model in several contexts. It is the interim inflation target in the policy rule, the expectational anchor for price determination and the benchmark for the evaluation of nominal interest rates in output determination. This triple role for \( L^m_t \) allows for substantial modelling simplification at the cost of some potential misspecification. Typically, policy rules involve longer-horizon inflation targets and inflation and output equations use shorter-horizon inflation expectations. We view our macro-finance specification as a parsimonious compromise that provides a useful description of term structure and macroeconomic dynamics.9

Finally, the specification of longer-term yields follows the standard no-arbitrage formulation described earlier for the yields-only model. The state space of the combined macro-finance model can be expressed by (1) with the re-definition of the state vector \( F_t \) to include output and inflation. The dynamic structure of this transition equation is determined by (10), (11), (12), (14) and (16). There are four structural shocks, \( \varepsilon_{\pi,t}, \varepsilon_{\gamma,t}, \varepsilon_{L,t} \) and \( \varepsilon_{S,t} \) which are assumed to be independently and normally distributed. The short rate is determined by (9), while yields of any other maturity are determined under the no-arbitrage assumption via (8). Specifically, for pricing longer-term bonds, the risk price associated with the structural shocks is assumed to be a linear function of just \( L^m_t \) and \( S^m_t \) and does not directly depend on the other state variables such as current or lagged \( \pi_t \) or \( y_t \). Such a risk specification, which relies solely on the latent factors \( L^m_t \) and \( S^m_t \) to determine interest-rate risk compensations, matches the yields-only formulation in Section 1 and other empirical finance research and allows comparison with earlier work.10 However, it should be stressed that the macroeconomic shocks \( \varepsilon_{\pi,t} \) and \( \varepsilon_{\gamma,t} \) are still able to affect the price of risk through their influence on \( \pi_t \) and \( y_t \) and therefore on the latent factors \( L^m_t \) and \( S^m_t \). The effect of macro shocks on yields will be discussed and illustrated below.

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9 Although the dynamic processes for \( \pi_t \) and \( L^m_t \) are very closely linked, a complete consistency between the perceived inflation target over the medium term is not enforced with the rational expectations of inflation from the inflation equation. Strictly enforcing such a consistency would greatly add to the computational burden.

10 Therefore, \( \lambda_1 \) continues to have just four non-zero entries, which greatly reduces the number of parameters to be estimated.

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2.2. Model Estimates

The above macro-finance model is estimated by maximum likelihood for the sample period from January 1988 to December 2000; see Rudebusch and Wu (2004) for complete details. Before examining the parameter estimates of the model, it is useful to compare the time series of $L^m_t$ and $S^m_t$ extracted from the estimated macro-finance model with the $L_t$ and $S_t$ extracted from the yields-only model. This is done in Figures 2 and 3. In both figures, the macro-finance model estimates of these factors (the solid lines) closely match the yields-only estimates (the dashed lines). Indeed, the two level factors have a correlation of 0.94, and the two slope factors have a correlation of 0.98. This close correspondence suggests that our macro-finance factors $L^m_t$ and $S^m_t$ can indeed be termed ‘level’ and ‘slope’ factors. Even more importantly, our macro-finance interpretation of these factors will have a direct bearing on the existing finance literature since we have obtained a very similar time series of factors. However, even though the historical evolution of $L^m_t$ and $S^m_t$ over the sample closely matches that of $L_t$ and $S_t$, the two sets of factors have notable differences in model dynamics and impulse responses, as discussed below.

Figures 2 and 3 also display connections between the latent factors and various macroeconomic variables, which provide some intuitive support for our analysis. Together with the level factors, Figure 2 displays the one-year-ahead expectation of annual inflation – which is from the Michigan survey of households – and an approximate measure of the 10-year-ahead inflation expectation measured as the spread between 10-year nominal and indexed Treasury debt. The

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11 This yield spread is only available starting in 1997 with the first issuance of US indexed debt and it is of course an imperfect and noisy measure of inflation expectations because of premiums for inflation risk and liquidity.
levels of the inflation rates and the latent factors are not comparable because the latter have been de-meaned. Still, the factors appear closely linked to expected inflation at both high and low frequencies. Over the entire sample, expected inflation and the level factor have all slowly trended down about 2 percentage points. This decline is consistent with the view that over this period the Federal Reserve conducted an opportunistic disinflation, with a gradual ratcheting down of inflation and the inflation target over time (Bomfim and Rudebusch, 2000). Month-by-month, the correlation between 1-year expected inflation and the macro-finance level factor is an impressive 0.73, which supports our contention of a close connection between the level factor and inflation. Together with the slope factors, Figure 3 also displays $y_t$, the measure of the output gap (or capacity utilisation). The correlation between the macro-finance slope factor and output is 0.66, which is consistent with the model implication that movements in slope capture the countercyclical monetary policy response of the central bank.

Table 2 reports the parameter estimates of the macro-finance model. First, consider the dynamics of the factors. The factor $L_m$ is very persistent, with a $\rho_L$ estimate of 0.989, which implies a small but significant weight on actual inflation. In contrast, the dynamics of $S_m$ in the macro-finance model can be given a very different interpretation than in the yields-only model. As evident in Figure 3, the slope factors are persistent in both models; however, in the macro-finance model, this persistence does not come from partial adjustment since the $\rho_S$ estimate is a minuscule 0.026. Instead, $S_m$ responds with only a very short lag to output and inflation. The persistence in $S_m$ reflects the fact that the Fed adjusts the short rate promptly to various determinants – output, inflation, and other influences in the residual $u_t$ – that are themselves quite persistent (e.g., $\rho_u = 0.975$). Thus, our estimate of $\rho_S$ decisively dismisses the interest rate smoothing or

12 Alternatively, our analysis considers only private sector perceptions and there may be differences between the true and perceived monetary policy inflation targets. Indeed, as shown in Orphanides and Williams (2005), when private agents have to learn about the monetary policy regime, long-run inflation expectations may drift quite far from even a constant true central bank inflation target.
monetary policy inertia interpretation of the persistence in the short rate. The persistent deviations of slope from fitted slope occur not because the Fed was slow to react to output and inflation but because the Fed responds to a variety of persistent determinants beyond current output and inflation. Some intuition for this result is given in the next subsection.

The monetary policy interpretation of the slope factor is supported by the values of the estimated inflation and output response coefficients, \( g_\pi \) and \( g_y \), which are 1.25 and 0.20, respectively. These estimates are similar to the usual single-equation estimates of the Taylor rule during this sample period (Rudebusch, 2002b). Overall, the macro-finance model estimates confirm a Taylor rule interpretation.

The estimated parameters describing the inflation dynamics also appear reasonable. In particular, the estimated weight on explicit forward-looking expectations in determining inflation, \( \mu_\pi \), is 0.074. Since this estimate is based on monthly data, with time aggregation, it implies a weight of about 0.21 on the interim inflation objective at a quarterly frequency. This estimate appears consistent with many earlier estimates obtained using a variety of different methods and specifications. For example, using survey data on expectations, Rudebusch (2002a) obtains a broadly comparable \( \mu_\pi \) estimate of 0.29, which is in the middle of the range of estimates in the literature. However, by using the yield curve to extract inflation expectations, our estimates bring new information to bear on this important macroeconomic question.

---

### Table 2

Macro-Finance Model Parameter Estimates

<table>
<thead>
<tr>
<th>Factor dynamics</th>
<th>( \rho_L )</th>
<th>(0.0068)</th>
<th>( g_\pi )</th>
<th>1.253</th>
<th>(0.0066)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_S )</td>
<td>0.026</td>
<td>(0.0111)</td>
<td>( g_y )</td>
<td>0.200</td>
<td>(0.0066)</td>
</tr>
<tr>
<td>( \rho_u )</td>
<td>0.975</td>
<td>(0.0062)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inflation dynamics</td>
<td>( \mu_\pi )</td>
<td>0.074</td>
<td>(0.0113)</td>
<td>( x_{s1} )</td>
<td>1.154</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>0.014</td>
<td>(0.0074)</td>
<td>( x_{s2} )</td>
<td>-0.155</td>
<td>(0.0066)</td>
</tr>
<tr>
<td>Output dynamics</td>
<td>( \beta_L )</td>
<td>0.099</td>
<td>(0.0066)</td>
<td>( \beta_{y1} )</td>
<td>0.918</td>
</tr>
<tr>
<td>( \beta_y )</td>
<td>0.089</td>
<td>(0.0067)</td>
<td>( \beta_{y2} )</td>
<td>0.078</td>
<td>(0.0066)</td>
</tr>
<tr>
<td>Risk price (( \lambda_1 ))</td>
<td>( L_t^m )</td>
<td></td>
<td>( S_t^m )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \Lambda_{L,t} )</td>
<td>-0.0045</td>
<td>(0.0068)</td>
<td>0.0168</td>
<td>(0.0068)</td>
<td></td>
</tr>
<tr>
<td>( \Lambda_{S,t} )</td>
<td>-0.0223</td>
<td>(0.0064)</td>
<td>0.0083</td>
<td>(0.0067)</td>
<td></td>
</tr>
<tr>
<td>Standard deviations</td>
<td>( \sigma_L )</td>
<td>0.342</td>
<td>(0.0089)</td>
<td>( \sigma_\pi )</td>
<td>0.238</td>
</tr>
<tr>
<td>( \sigma_S )</td>
<td>0.559</td>
<td>(0.0313)</td>
<td>( \sigma_y )</td>
<td>0.603</td>
<td>(0.0128)</td>
</tr>
<tr>
<td>Standard deviations of measurement error</td>
<td>5-month</td>
<td>0.288</td>
<td>(0.0162)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>12-month</td>
<td>0.334</td>
<td>(0.0194)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>36-month</td>
<td>0.127</td>
<td>(0.0094)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note. Standard errors of the estimates are in parentheses.
The estimated parameters describing the output dynamics also fall within reasonable ranges. Specifically, the estimated value of $\mu_y = 0.009$ implies a negligible weight at a quarterly frequency on forward-looking output expectations in the determination of output behaviour. This is very much in accord with the maximum likelihood estimation results reported by Fuhrer and Rudebusch (2004).

The risk price matrix ($\lambda_1$) appears significant, and the model fits the 3-month, 12-month, and 36-month yields with measurement error standard deviations that are quite comparable to the yields-only model.

2.3. Analysis of Dynamics

The dynamics of the estimated macro-finance model are quite interesting and intuitive. First, consider the instantaneous responses of the yield curve to a positive shock in $L_m^t$ or $S_m^t$. These responses, which are analogous to those in Figure 1 for the yields-only model, are displayed in Figure 4. As is clear from the structure of the factor dynamics above, a shock to $L_m^t$ has two very different effects on the short rate $i_t$. First, it directly raises the short rate one-for-one according to (9). Second, from (11), an increase in $L_m^t$ reduces $S_m^t$ and pushes down the short rate by more than one-for-one – given the estimate of $g_p = 1.20$. The macroeconomic interpretation of this latter effect is that an increase in the perceived inflation target must be associated with an easing of monetary conditions so inflation can rise to its new target. Given some persistence in inflation, easier monetary conditions (lower real rates) require an initial decline in the short-

![Fig 4. Initial Yield Curve Response to Level and Slope Shocks in Macro-Finance Model](image)

Note: These curves show the impact response from a 1 percentage point increase in level or slope on the yield of a given maturity.

14 The interest rate sensitivity of output ($\beta_y = 0.089$), after taking into account the time aggregation and the use of capacity utilisation rather than the output gap, appears broadly in line with estimates that use the entire postwar sample of quarterly data.

15 In a model without nominal rigidities or persistence, inflation would simply jump to the new target. Such a model, with $\mu_e = 1$, does not appear to fit the data.
term nominal interest rate. This second effect dominates at the short end, so a positive shock to $L_t^m$ (the perceived inflation objective) initially lowers short-term yields. However, at intermediate- and long-term maturities, the first effect dominates, and the increase in $L_t^m$ raises the yields one-for-one, as in the yields-only model. Therefore, the initial effect of an increase in $L_t^m$ is not quite a parallel shift of the yield curve, but rather a tilt upward. The initial response of the yield curve to a positive shock in $S_t^m$ is similar to the one shown in Figure 1 for the yields-only model. A positive shock to $S_t^m$ (specifically to $\epsilon_{S,t}$) increases short-term bond yields but has progressively less effect on bonds of greater maturity. Thus, the positive shock initially decreases the slope of the yield curve and produces a tilt downward.16

At first glance, Figure 4 may appear somewhat inconsistent with Figures 1 and 2. Specifically, although $L_t^m$ and $L_t$ appear to move together over the historical sample (Figure 2), their impulses induce different initial responses in the yield curve (as a comparison of Figures 1 and 4 illustrates). However, a reconciliation of these results is suggested by examining a ‘rotation’ or re-definition of the state space of the yields-only model that is observationally equivalent to the original yields-only model. There is a rotated yields-only model that can closely match both the factor loadings of the macro-finance model as well as the historical behaviour of the level and slope factors in the original yields-only model and in the macro-finance model. That is, the estimated paths of level and slope are largely insensitive to the exact form of the factor loadings; see Rudebusch and Wu (2004) for details.

In addition, as is clear from (10), the macro-finance model is parsing out some of the traditional level shock effect to inflation. Indeed, similar to Figure 4, the solid lines in

![Fig. 5. Initial Yield Curve Response to Output and Inflation Shocks](image)

**Note:** The solid lines show the impact responses on the yield curve from a 1 percentage point increase in inflation or output in the estimated macro-finance model. The dashed lines give similar responses in a macro-finance model that assumes substantial monetary policy inertia ($\rho_s = 0.9$) and serially uncorrelated policy shocks ($\rho_u = 0$).

16 Figure 4 suggests that $L_t^m$ and $S_t^m$ might be better labelled ‘Long-term’ and ‘Short-term’ factors because those are the locations of maximum influence; however, we will continue using the standard terminology.

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Figure 5 display the initial responses of the yield curve to inflation and output shocks in the estimated macro-finance model. Positive shocks to inflation and output in this model are followed by immediate increases in short-term interest rates, and for the inflation shock, these increases are more than one-for-one. These quick responses reflect the absence of monetary policy partial adjustment or inertia (the estimated $\rho_S = 0.026$). In contrast, the dashed lines in Figure 5 display the yield curve responses from a model that is identical to the estimated macro-finance model except that $\rho_S$ is set equal to 0.9 and $\rho_u$ equals 0. This hypothetical alternative model has substantial monetary policy inertia and it displays markedly weaker responses to inflation and output shocks by yields that have maturities less than two years. The two quite different responses of the yield curve in these models illustrate the potential importance of the information from the term structure for discriminating between the two models. Given the system estimates, it is clear that the data prefer the macro-finance model without policy inertia.

Now consider the dynamics of the macro-finance model more generally. Figures 6 and 7 display the impulse responses of the macroeconomic variables and bond yields to a one standard deviation increase in each of the four structural shocks in the model. Each response is measured as a percentage point deviation from the steady state.

**Fig. 6. Impulse Responses to Macro Shocks in Macro-Finance Model**

*Note:* All responses are percentage point deviations from baseline. The time scale is in months.

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Figure 6 focuses on the macro shocks, and the first column shows the impulse responses to an inflation shock. Such a shock leads to an instant 25-basis-point increase in the inflation rate, which is gradually reversed over the next two years. Inflation does not, however, return to its original level because the sustained period of higher inflation boosts perceptions of the underlying inflation target $L_t^m$. The initial jump in inflation also induces a tightening of monetary policy that raises the slope factor and short-term interest rates. Indeed, the 1-month rate first jumps about 30 basis points and then gradually falls. The 12-month and 5-year yields also increase in response to the inflation shock but by smaller amounts. The higher interest rates lead to a gradual decrease in output, which damps inflation.

The second column of Figure 6 displays the impulse responses to a positive output shock, which increases capacity utilisation by 0.6 percentage points. The higher output gradually boosts inflation and, in response to higher output and inflation, the central bank increases the slope factor and interest rates. In contrast to the differential interest rate responses in the first column, all of the interest rates in the second column show fairly similar increases. The bond yields of all maturities are still approximately 5 basis points higher than their initial levels even 5 years after the shock, because some of the rise in inflation has passed through to the perceived inflation target $L_t^m$.

One particularly noteworthy feature of the responses in Figure 6 is how long-term interest rates respond to macroeconomic shocks. As stressed by Gürkaynak et al. (2005), long rates do appear empirically to respond to news about macroeconomic variables; however, standard macroeconomic models generally cannot reproduce such movements because their variables revert to the steady state too quickly. By allowing for time variation in the inflation target, the macro-finance model can generate long-lasting macro effects and hence long rates that do respond to the macro shocks.

Figure 7 provides the responses of the variables to perceived changes in monetary policy. There are two types of such policy changes to consider, as in Ellingsen and Söderström (2001): namely, changes in policy preferences and changes in macroeconomic policy determinants. In the macro-finance model, the first is a perceived shift in the inflation target or level factor. The first column displays the impulse responses to such a level shock, which increases the inflation target by 34 basis points – essentially on a permanent basis. In order to push inflation up to this higher target, the monetary authority must ease rates, so the slope factor and the 1-month rate fall immediately after the level shock. The short rate then gradually rises to a long-run average that essentially matches the increase in the inflation target. The 12-month rate reaches the new long-run level more quickly and the 5-year yield jumps up to that level immediately. The easing of monetary policy in real terms boosts output and inflation. Inflation converges to the new inflation target but output returns to about its initial level.

The second column of Figure 7 displays the response to a slope shock, which is the second type of policy change: a perceived policy response to some development in the economy – other than current output and inflation – such as a credit crunch reflecting

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17 Such a shift could reflect the imperfect transparency of an unchanged actual inflation goal in the US or its imperfect credibility. Overall then, in important respects, this analysis improves on the usual monetary VAR, which contains a flawed specification of monetary policy (Rudebusch, 1998). In particular, the use of level, slope and the funds rate allows a much more subtle and flexible description of monetary policy.
balance sheet problems in the banking system, an asset price misalignment relative to fundamentals, or a development in foreign economies abroad.\textsuperscript{18} A one standard deviation slope shock raises the 1-month interest rate by 56 basis points and raises the 12-month and 5-year yields by 42 and 5 basis points, respectively. Furthermore, in response to this persistent slope shock, interest rates remain elevated for a considerable period of time. Given this sustained tighter monetary policy, the capacity utilisation rate gradually declines and remains well below its initial level for several years, generating a decline in inflation as well. Falling inflation translates into perceptions of a declining inflation target, which eventually helps to push all interest rates to fall below their initial values. The effects on output and inflation are very persistent because of the long-lasting nature of the slope shock; see Rudebusch and Wu (2004) for variance decompositions.

\textsuperscript{18} For example, Federal Reserve Governor Laurence Meyer (1999) described the easing of policy during late 1998: ‘There are three developments, each of which, I believe, contributed to this decline in the funds rate relative to Taylor Rule prescription. The first event was the dramatic financial market turbulence, following the Russian default and devaluation. The decline in the federal funds rate was, in my view, appropriate to offset the sharp deterioration in financial market conditions, including wider private risk spreads, evidence of tighter underwriting and loan terms at banks, and sharply reduced liquidity in financial markets.’

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3. Conclusion

This article contributes to what is now a burgeoning macro-finance term structure literature (Diebold et al., 2005) and it is perhaps useful to conclude with an overview of the general strengths and weaknesses of combining finance and macroeconomic descriptions of the yield curve.

Foremost among the strengths of such a combination are the broad interdisciplinary insights that can be provided on a range of important topics. For example, with a combined macro-finance framework, this article was able to describe the macroeconomic underpinnings of movements in the yield curve. In particular, it characterises the relationships between the no-arbitrage latent term structure factors prevalent in the finance literature and various macroeconomic variables. The level factor is linked to the perceived medium-term central bank inflation target and the slope factor is related to cyclical variation in inflation and output gaps as the central bank moves the short rate in order to achieve its macroeconomic policy goals. The above analysis also has several interesting implications for the macroeconomics literature. Notably, the term structure evidence, even after allowing for time-varying term premiums, is able to shed light on the amount of partial adjustment in the setting of monetary policy, which appears to be negligible. Also, new information is drawn from the yield curve on the issue of the importance of expectations in the determination of output and inflation.

This macro-finance model can also address other questions. In Rudebusch and Wu (2007), we use it to examine how the dynamics of the term structure and interest rate risk may have changed over time. Specifically, we explore the links between recent shifts in the behaviour of the term structure and the recent ‘Great Moderation’ in fluctuations of the US economy. If one assumes that the factors underlying changes in the macro dynamics have also left their imprint on yield curve dynamics, then the macro-finance perspective helps to illuminate the nature of these changes. Indeed, it appears that the bond market’s assessment of risk associated with inflation has shifted in a way that suggests the recent macroeconomic stabilisation appears linked to a change in the monetary policy environment. In Rudebusch et al. (2006), we use two macro-finance models to examine the ‘conundrum’ of surprisingly low long-term US bond yields during the 2004 and 2005 episode of tightening monetary policy. When viewed through a macro-finance lens, the extent of this conundrum is clear although much of its exact source remains unexplained. Two other topics for likely fruitful future analysis with a macro-finance approach include fiscal policy, where these models may be powerful tools to evaluate government debt issuance and management, and business cycle forecasting, where these models could help to interpret the significant power of the term spread in predicting recessions described in, for example, Rudebusch and Williams (2007).

Although the macro-finance combination is a powerful tool that can potentially illuminate many issues, there are also theoretical and practical weaknesses to this approach as currently implemented that should be noted. First, we should stress that the macro-finance term structure literature is in its infancy and, like many others in this literature, our model is an intermediate step between an atheoretical empirical representation and a deep theoretical one. We think such intermediate models are useful and interesting but there is also a desire to obtain a complete specification in terms of...
underlying preference and technology parameters. However, the affine term structure models in the finance literature are largely statistical in nature and the nominal discount factor is not expressed in terms of marginal utility and inflation. In particular, the finance literature has not yet justified – in terms of underlying preferences – the popular affine risk-price representation that we adopt. This state-dependent specification of the market price of risk may seem ad hoc but it is widely used in term structure studies because it performs well in matching certain empirical properties (Dai and Singleton, 2002). One of the goals of our study is to provide a macroeconomic interpretation of the term structure factors found in this existing finance literature; thus, we adopt the usual affine risk-price specification of that literature. Ideally, of course, the representation of the price of risk would be consistent with the preferences and technology underpinning the output and inflation equations. However, just as the link between the utility function and the price of risk is implicit in the affine specification, there is only a tacit link between the utility function and our macro specification (for example, the macro data include capital goods and net exports and aggregation over heterogeneous agents). That is, we have joined together an affine term structure model with a hybrid New Keynesian model, each of which have empirical successes in their respective areas but also have somewhat tenuous theoretical foundations. The goal of linking risk pricing, interest rates, output, and inflation together in a deep theoretical specification of preferences and technology (so the asset pricing kernel is consistent with the macrodynamics) and still retaining an empirical fit remains a major challenge for the future. The linkage between the representations of risk and the macro structure in a rigorous general equilibrium model has only been formulated for very simple models and has not been very successful (Wu, 2006; Rudebusch et al., 2007). Obtaining a consistent non-trivial macro-finance pricing kernel remains a key area for future research.

There are also a variety of other macro-finance model specification issues to examine. Many believe that long-maturity interest rates (with embedded term premiums) are a direct determinant of output (Rudebusch et al., 2007). As a theoretical matter, this link could be motivated by considering financial frictions but, as noted above, such a formulation would be difficult to estimate. Similarly, typical macro-finance specifications of (perceived) monetary policy remain quite rudimentary. For example, the specification linking the level factor to inflation in our model is overly mechanical, since financial market participants in fact are undoubtedly conducting a subtle filtering of the available data to obtain underlying inflation objectives. Also, an elaboration of the policy response to include real-time data and a forward-looking perspective would seem needed.

Finally, there are important practical computational roadblocks that should be noted in implementing the macro-finance term structure models. Even the pure finance term structure models are very difficult to estimate, plagued by multiple local optima, imprecise parameter estimates, and unknown small-sample distributional properties (Christensen et al., 2007). These difficulties are typically magnified when adding the greater complexity of macroeconomic interactions. For many finance researchers, the additional computational cost of adding serious macroeconomic relationships may be too high. Similarly, for many macro researchers, the cost of modelling time varying term premiums may also be too steep. Although the computationally intensive nature
of the estimation is a significant drawback, we think the macro-finance combination modelling strategy is still a powerful tool that can potentially illuminate many issues.

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References


