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Using a long-term interest rate as the monetary policy instrument ☆

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Abstract

Using a short-term interest rate as the monetary policy instrument can be problematic near its zero bound constraint. An alternative strategy is to use a long-term interest rate as the policy instrument. We find when Taylor-type policy rules are used by the central bank to set the long rate in a standard New Keynesian model, indeterminacy—that is, multiple rational expectations equilibria—may often result. However, a policy rule with a long-rate policy instrument that responds in a "forward-looking" fashion to inflation expectations can avoid the problem of indeterminacy.

JEL classification: D83; D84; E32; E43; E52

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> [Fed Chairman] Greenspan assured the congressional Joint Economic Committee that even with the Fed's key economic policy lever, the federal funds rate, at a 41-year low of 1.25 percent, the central bank has other resources to influence interest rates to jump-start economic growth. He said that in addition to pushing the funds rate, the interest that banks charge each other on overnight loans, closer to zero, the Fed can simply begin buying longer-term Treasury securities to drive longer-term interest rates lower.
>
> Associated Press Newswires, May 21, 2003

1. Introduction

Over the past decade, inflation rates in many countries have fallen to levels not much above zero. This long-sought return to price stability should provide significant benefits in terms of enhanced economic efficiency and performance; however, as noted early on by Summers (1991), low inflation rates may also make it harder for central banks to achieve their macroeconomic stabilization goals. Specifically, as inflation has declined, short-term nominal interest rates, which are the usual instrument of monetary policy, also have fallen and have closed in on, or even run up against, their lower bound of zero, likely limiting the extent to which real interest rates can be lowered. This constraint may diminish the ability of a central bank to stimulate the economy and offset adverse macroeconomic shocks through the usual policy transmission mechanism of lower real rates (see Reifschneider and Williams, 2000 and Clouse et al., 2003).

This long-standing theoretical concern about the so-called liquidity trap has been given a visceral immediacy by the example of Japan, which spent more than a decade mired in economic stagnation and deflation. In trying to stimulate the Japanese economy, the Bank of Japan came up against the zero bound when it lowered its policy rate (the overnight call rate) to zero in February 1999, and further policy stimulus via lower short rates was clearly impossible. Japan's predicament generated much discussion about the nature of the zero bound constraint and its importance in hindering macroeconomic stabilization. In response, many researchers have proposed using a variety of alternative monetary policy strategies and policy instruments other than the short rate—such as the monetary base, a long-term interest rate, or the exchange rate—to provide increased stimulus in such a situation (notably, Krugman, 1998; Meltzer, 2001; Svensson, 2001; McCallum, 2002; Okina and Shiratsuka, 2004).

These alternative monetary policy proposals have also been discussed for the U.S. economy. As illustrated by the press coverage of Chairman Greenspan's testimony above, faced with sluggish real growth in 2003 and inflation and short-term interest rates at historic lows, the Federal Reserve also studied various strategies to stimulate the economy if short rates fell to their lower bound (e.g., Bernanke and Reinhart, 2004). A common thread in these alternative policy strategies to stimulate the economy is their reliance on influencing the public's expectations of future policy actions—often by committing to a clear target path for some future economic variable, such as the short-term policy rate, the price level, or the currency exchange rate.[1] The forward-looking nature of these alternative policy proposals is perhaps most transparent for the case of influencing expected future short-term interest rates, and thereby long-term bond rates. Accordingly, the first line of defense at the zero bound appears to be flattening the entire yield curve.

Although flattening the yield curve is the clear objective of policy when confronting the zero bound, it is not obvious that market expectations of future short rates will automatically align themselves with those desired by the central bank. The research mentioned above assumes that the central bank can credibly commit to future policy actions and that expectations are rational. Under these assumptions, the central bank simply needs to announce an appropriate policy rule, sit back, and be confident that markets will coordinate on the desired equilibrium. In such an environment with credibility and appropriate policy, the zero bound has little effect. In practice, however, one may not have full confidence in the seamless communication of future policy actions to markets imagined in theoretical models. Indeed, this concern that markets may misinterpret the central bank's intentions is particularly acute at the zero bound when it cannot use the short rate to "back up its words with actions."

Instead, it may be useful, or even necessary, to signal to markets the desired path of expected short rates through concrete actions. In particular, the central bank can transmit its intent by using the long-term bond rate as the policy instrument. The goal of such an alternative policy approach is to effectively communicate to markets a path for future interest rates, as summarized by the targeted long rate, that replicates as closely as possible the desired equilibrium implied by the optimal policy under commitment. This approach to "targeting" long rates is a type of "targeting rule" in the sense of Lars Svensson, by which the central bank is committed to achieving a relationship between long rates and other endogenous variables.[2] Therefore, consistent with Chairman Greenspan's views, after the short rate is pushed to zero, an obvious next step for monetary stimulus is to push interest rates of progressively longer maturities to zero as well.

Despite being a natural alternative to a short-rate policy instrument, there has been remarkably little analysis of the properties of a long-rate policy rule.[3] This is somewhat surprising because affecting the economy through long rates has typically been the acknowledged channel for monetary policy transmission—the overnight rate on its own is not an important factor on the economic decisions of most consumers and businesses. That is, a long-rate policy may be useful strategy even in cases where the zero bound did not constrain policy. Therefore, in Sections 2 and 3, we illustrate how minor the step is in moving from a short-rate to a long-rate policy

[1]See, for example, Reifschneider and Williams (2000), Svensson (2001), McCallum (2002), and Eggertsson and Woodford (2003a,b).

[2]Our approach is in the spirit of Svensson (2001), who favors foreign exchange intervention to coordinate expectations on the desired future path of policy.

[3]Kulish (2004), which is contemporaneous but independent to our own analysis, is an interesting exception.

instrument—at least on a formal modeling level. Section 2 analyzes the standard formulation of a New Keynesian macroeconomic model coupled with a Taylor-type rule for setting the short-term interest rate as the monetary policy instrument, and in Section 3, the one small modification made is that the rate on a longer-maturity bond becomes the policy instrument.

However "obvious" it may seem to shift from using a short rate to a long rate as the monetary policy instrument, we find that doing so raises the important issue of indeterminacy. Indeterminacy arises because many possible future paths for the short rate may be consistent with a given setting of the long rate. Thus, using the long rate as a monetary policy instrument increases the likelihood of indeterminacy even for policy rules that appear prima facie to be reasonable. One solution to the potential indeterminacy is for the central back to have a clear signaling and communications strategy about the intended path for the short rate that is associated with the long-rate monetary policy rule. An important complementary factor is the stability of the rational expectations equilibrium (REE) under learning, which indicates whether atomistic economic agents could coordinate expectations on a candidate REE (Evans and Honkapohja, 2001). Clearly, if an equilibrium is stable under learning, such learning could help reinforce a central bank's communication strategy. In addition, in the case of multiple equilibria, stability under learning can be used as an equilibrium selection criterion.

Section 4 extends the analysis to explore an alternative specification of the monetary policy rule that follows McCallum's recommendation that the determinants of an operational policy must be observable by the policymaker in real time. We find that policies that respond to observable lagged data are also subject to the problem of indeterminacy and that this problem is even more profound owing to the possibility of multiple minimum state variable (MSV) solutions.

Given the risk of indeterminacy with standard rule formulations, Section 5 examines an alternative specification of the long-rate policy rule that yields a determinate outcome. Specifically, we show that "forward-looking" versions of a policy rule that uses the bond rate as the policy instrument entirely overcome the problems of indeterminacy that plague rules responding only to contemporaneous or lagged variables. Our finding that properly specified forecast-based long-rate policies guarantee a determinate REE while backward-looking variants do not stands in stark contrast to the literature on short-rate policy instruments, where the opposite conclusion is often found to be true (e.g., Bernanke and Woodford, 1997).

Finally, as noted in Section 6, although our analysis focuses only on the long rate, our results appear relevant for other proposals for avoiding deflation that involve shaping the path of expectations about the future.

2. Analysis of a short-rate policy instrument

This section introduces the basic framework for our analysis, which includes a stylized New Keynesian model. This section also examines the conditions for determinacy and learnability under the standard assumption that the monetary policy instrument is the short-term nominal interest rate. These results provide a benchmark for comparison in the next section, which investigates the effects of using a long-rate policy instrument.

2.1. Conditions for determinacy

We consider the canonical forward-looking IS-AS model (e.g., Walsh, 2003 or Woodford, 2003):

$$y_t = -\beta(i_t - \mathrm{E}_t\pi_{t+1}) + \mathrm{E}_t y_{t+1} + e_t, \tag{1}$$

$$\pi_t = \mathrm{E}_t\pi_{t+1} + \alpha y_t + u_t, \tag{2}$$

where y_t denotes the output gap (the percent deviation of output from its natural rate), π_t is the inflation rate, i_t is the short-term interest rate, and α, $\beta > 0$. For convenience, we assume that the discount rate that would normally appear in the equation describing inflation is unity, and that the natural rate of interest is zero. The innovations e_t and u_t are assumed to be white noise.[4]

We start by assuming that monetary policy is implemented through a Taylor-type policy rule that takes the form[5]

$$i_t = \gamma\pi_t. \tag{3}$$

We implicitly assume that the target inflation rate is zero. Substituting the policy rule into the IS equation, we can rewrite the system:

$$\begin{pmatrix} 1 & \beta\gamma \\ -\alpha & 1 \end{pmatrix}\begin{pmatrix} y_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} 1 & \beta \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \mathrm{E}_t y_{t+1} \\ \mathrm{E}_t\pi_{t+1} \end{pmatrix} + \begin{pmatrix} e_t \\ u_t \end{pmatrix}, \tag{4}$$

or, with the appropriate identifications,

$$Hx_t = F\mathrm{E}_t x_{t+1} + \varepsilon_t,$$

where $x_t \equiv (y_t, \pi_t)'$ and $\varepsilon_t \equiv (e_t, u_t)'$.

A linear model is said to be determinate under a specific policy rule if a unique non-explosive REE exists, indeterminate if multiple REE exist, and explosive if no such REE exist (see Evans and McGough, 2003 for details). In the analysis of determinacy, it is useful to define the forecast error vector, $\xi_t = x_t - \mathrm{E}_{t-1}x_t$. Taking advantage of the fact that the matrix F is invertible and after some substitutions, we obtain a dynamic equation describing the system in terms of fundamentals and forecast errors:

$$x_t = F^{-1}Hx_{t-1} - F^{-1}\varepsilon_{t-1} + \xi_t.$$

[4] In preliminary work, we have obtained broadly similar results to those below in hybrid models that have output and inflation dynamics with both forward- and backward-looking elements.

[5] The inclusion of an output gap response in the policy rule would change little but would make getting analytical results more difficult.

Stacking x and ε, we have

$$\begin{pmatrix} x_t \\ \varepsilon_t \end{pmatrix} = \begin{pmatrix} F^{-1}H & -F^{-1} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ \varepsilon_{t-1} \end{pmatrix} + \begin{pmatrix} \xi_t \\ \varepsilon_t \end{pmatrix}. \tag{5}$$

Let A denote the leading matrix on the right-hand side of this equation.

We now analyze the properties of this system. This model has two free variables and will thus be determinate provided that the two eigenvalues of $F^{-1}H$ are of modulus greater than one. The condition for determinacy in this model with the specified short-rate monetary policy rule is a version of the so-called Taylor principle, according to which the nominal rate must be increased more than one-for-one in response to an increase in the inflation rate, as stated in the following proposition.

Proposition 1. *The model is determinate if* $\gamma > 1$ *and indeterminate otherwise.*

Refer to the appendix for proofs of all propositions.

In the determinate case, the equilibrium may be computed by decoupling the dynamics of the system along the eigenspaces by diagonalizing $A = S\Lambda S^{-1}$ and defining $z_t \equiv S^{-1}(x_t', \varepsilon_t')'$. Let $S^{ij} \equiv (S^{-1})_{ij}$ and

$$S_k^{ij} \equiv \begin{pmatrix} S^{ij} & S^{ij+1} \\ S^{kj} & S^{kj+1} \end{pmatrix}.$$

Then, rewriting Eq. (5) yields

$$z_t = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & 0 \end{pmatrix} z_{t-1} + S^{-1} \begin{pmatrix} \xi_t \\ \varepsilon_t \end{pmatrix}, \tag{6}$$

where the λ_i are the eigenvalues, which are labeled here and subsequently in descending order according to modulus; i.e., $|\lambda_1| \geqslant |\lambda_2| \geqslant \cdots \geqslant |\lambda_n|$. Choosing ξ_t to assure that $z_{it} = 0$ for $i = 1, 2$ yields a unique REE of the form

$$x_t = -(S_2^{11})^{-1} S_2^{13} \varepsilon_t. \tag{7}$$

Note that in this model with the specified policy rule, the REE is a white noise process.

2.2. Stability under learning

We use expectational stability as our criterion for judging whether agents may be able to coordinate on specific equilibria. This is because, for a wide range of models and solutions, E-stability has been shown to govern the local stability of rational expectations equilibria under least-squares learning. In many cases this correspondence can be proved, and in cases where this cannot be formally demonstrated the "E-stability principle" has been validated through simulations. Before giving details, we provide an overview of E-stability; for further reading see Evans and Honkapohja (2001).

The model at hand may be written in reduced form as follows:

$$x_t = H^{-1}F\mathrm{E}_t^* x_{t+1} + H^{-1}\varepsilon_t. \tag{8}$$

We now write $\mathrm{E}_t^* x_{t+1}$ to indicate that we no longer impose rational expectations, and at issue is how agents form their time t expectations E_t^*. Backing away from the benchmark that agents are fully rational, we assume that agents believe the endogenous variable x_t depends linearly on a constant, lagged endogenous variables, current exogenous shocks, and possibly exogenous sunspots. Combining these regressors into the vector X_t, we postulate a perceived law of motion (PLM) $x_t = \Theta' X_t$. Agents then use this PLM to form their expectations of x_{t+1}.

Under real-time learning agents estimate Θ using an algorithm such as recursive least-squares and these estimates are updated over time. Given a particular value for Θ the corresponding expectations $\mathrm{E}_t^* x_{t+1}$ can be computed, the expectations can be substituted in the reduced form equation above, and the true data generating process, or actual law of motion (ALM), thus determined. If the PLM is well specified then the ALM will have the same form: $x_t = T(\Theta)' X_t$. In particular, the ALM will depend linearly on the same variables as did the PLM. Thus a map, known as the T-map, is constructed, taking the perceived parameters to the implied parameters. A fixed point of this map defines a REE.

Once the T-map is obtained, stability under learning can be addressed as follows. Let the equilibrium be characterized by the fixed point Θ^*, and consider the differential equation

$$\frac{\mathrm{d}\Theta}{\mathrm{d}\tau} = T(\Theta) - \Theta. \tag{9}$$

Note that Θ^* is a rest point of this ordinary differential equation. The equilibrium corresponding to the fixed point is said to be *E-stable* if it is a locally asymptotically stable fixed point of (9). The E-stability principle tells us that E-stable equilibria are locally learnable for least squares and closely related algorithms. That is, if Θ_t is the time t estimate of the coefficient vector Θ, and if Θ_t is updated over time using recursive least squares, then Θ^* is a possible convergence point, i.e. locally $\Theta_t \to \Theta^*$ if and only if Θ^* is E-stable. The intuition behind this principle is that a reasonable learning algorithm, such as least squares, gradually adjusts estimates Θ_t in the direction of the actual parameters $T(\Theta_t)$ that are generating the data. For an E-stable fixed point Θ^* such a procedure would then be expected to converge locally.

The above discussion has implicitly assumed a rest point Θ^* that is locally isolated. In this case it is locally asymptotically stable under (9) provided all eigenvalues of the Jacobian of T at Θ^* have real parts less than one, and it is unstable if the Jacobian has at least one eigenvalue with real part greater than one. Because we are studying sunspot equilibria, the set of rest points of (9) may have unbounded continua as connected components. Along these components the T-map will always be neutrally stable, and thus will have at least one eigenvalue equal to unity.[6] In this case we say a sunspot equilibrium is E-stable if the Jacobian of the T-map has eigenvalues with

[6]The number of unit eigenvalues will be equal to the dimension of these components.

real part less than one, apart from unit eigenvalues arising from the equilibrium connected components.

We now turn to the specifics of stability analysis in our model. To make forecasts, agents are assumed to estimate a PLM of the form $x_t = a + b\varepsilon_t$. The implied T-map is given by

$$a \longrightarrow H^{-1}Fa,$$
$$b \longrightarrow H^{-1} = -(S_2^{11})^{-1}S_2^{13}.$$

Convergence in b obtains trivially, and convergence in a obtains provided the real parts of the eigenvalues of $H^{-1}F$ are less than one, which, in this particular model, obtains if the policy yields a determinate REE, as stated in the following proposition.

Proposition 2. *If* $\gamma > 1$, *then the unique REE is E-stable.*

This proposition is a simple consequence of the determinacy of the model depending on the eigenvalues of $(H^{-1}F)^{-1}$.

2.3. Long-term bond rates

In preparation of the analysis of monetary policy rules that use a longer-maturity interest rate as the policy instrument, we now introduce multi-period bond rates to the model. Following Shiller (1979), let the n-period bond rate (for $1 < n < \infty$) be given by

$$i_{n,t} = \left(\sum_{j=0}^{n-1} \delta^j\right)^{-1} \mathrm{E}_t \sum_{j=0}^{n-1} \delta^j i_{t+j} + \Psi_{n,t}, \tag{10}$$

where the parameter $0 < \delta \leqslant 1$ is a constant that is unity for discount (zero coupon) bonds, but less than unity for coupon bonds (and equal to the inverse of the gross discount rate), and $\Psi_{n,t}$ is a term or risk premium.[7]

Assuming that the value of γ in the monetary policy rule (3) yields a unique stable equilibrium, we can substitute future expected short-term interest rates into Eq. (10), yielding the reduced-form equation for the n-period bond rate. From above, we know that expected inflation in future periods is zero, and, thus, so are expected short rates. Therefore, the reduced form equation describing rates on longer-maturity bonds is given by:

$$i_{n,t} = G_n(\gamma)\pi_t \equiv \left(\sum_{j=0}^{n-1} \delta^j\right)^{-1} \gamma\pi_t + \Psi_{n,t}. \tag{11}$$

[7] Rudebusch and Wu (2004) discuss theoretical and empirical aspects of this equation and the time-varying term premium from a modern asset pricing perspective.

3. Analysis of a long-rate policy instrument

Section 2 provided a canonical analysis of a short-rate policy rule in a New Keynesian model. The only nonstandard element was the addition of a long rate that was extraneous to the system. We now analyze, after providing some introductory motivation, the determinacy and stability properties of that same model—except with the long rate specified as the monetary policy instrument.

3.1. Motivation for a long-rate policy instrument

When reduced to simplest terms, two critical features of the framework in Section 2 make it amenable to monetary policy analysis: first, the central bank can control the short-term interest rate; second, movements in the short rate affect output and inflation. Similarly, the use of a long rate as the monetary policy instrument raises two fundamental questions: (1) Can the monetary authority control the long rate? (2) Do movements in the long rate affect the economy? Here, as a precursor to our formal analysis, we provide some discussion of these issues.

Regarding central bank control of the long rate, it is useful to consider the decomposition of the long rate into expected future short rates and the term premium, as given in Eq. (10). The central bank can clearly control the current short rate and therefore also the short rate at each point in time in the future. Whether this control of the *actual* path of future short rates translates into control of the *expected* path of future short rates, and hence the long rate, depends on the extent to which the public understands and believes that the central bank will deliver on future promised actions. As stressed by Bernanke and Reinhart (2004), clear communication of the central bank's commitment to a future interest rate path is a crucial element in obtaining this understanding.[8] In addition, the credibility of such a commitment can likely be enhanced by taking action in several ways. For example, the central bank can engage in open market sales or purchases of long bonds directly or take positions in options and futures markets that indicate a certain path of future short rates. Such actions can provide a signal that causes private agents to expect a particular short-rate policy path (e.g., Clouse et al., 2003).[9]

[8]Bernanke et al. (2004) provide empirical evidence of the efficacy of central bank communication in shaping expectations of the future path of the funds rate. Specifically, the statements issued after meetings of the Federal Open Market Committee appear to have important effects on interest rate futures.

[9]One issue often raised in discussions of directly affecting prices of long-term bonds is the central bank's exposure to capital losses. Of course, the central bank's portfolio is always subject to capital gains and losses resulting from movements in market prices of long-term bonds. We view any intervention in long-term bond markets as primarily focused on coordinating market expectations of future short rates on the appropriate policy path; therefore, the capital gains and losses would be the same as when expectations were aligned with the central bank's intentions in the first place. If, however, the central bank attempted to substantially reduce the term premium through its actions, an experiment that we do not consider in this paper, its exposure to capital losses would increase. Nonetheless, it is unclear whether such capital losses should be a salient concern for the design of policy.

Many researchers have also pointed to the term premium, $\Psi_{n,t}$, as a second channel through which a central bank may be able to alter the long bond rate (e.g., Goodfriend, 2000; Clouse et al., 2003).

In particular, as noted by Bernanke and Reinhart (2004), altering the composition of the central bank's balance sheet—shifting, say, from holding shorter to longer maturity debt—may affect term premiums if investors view the assets as imperfect substitutes. Whether such portfolio rebalancing effects could be usefully exploited is difficult to ascertain, and we do not rely on this somewhat speculative channel in our analysis.[10]

We therefore assume that the central bank can use the long-term rate as a policy instrument, and now turn to the question of whether changes in the long rate affect the economy, and here the decomposition in Eq. (10) again is useful. First, consider a change in the long rate arising from a change in the expected path of future short rates. The effect of this change on aggregate demand can be seen by iterating Eq. (1) forward n periods to obtain

$$y_t = -\beta \mathrm{E}_t \sum_{j=0}^{n-1} (i_{t+j} - \pi_{t+1+j}) + \mathrm{E}_t y_{t+n} + e_t. \tag{12}$$

That is, the intertemporal substitution in Eq. (1) implies that the *path* of expected future short rates matters for aggregate demand, and, according to the theory of asset pricing, Eq. (10), that path is embedded in long rates. Therefore, there is a direct link between the long rate and aggregate demand through short-rate expectations.

The model of intertemporal substitution, however, also raises the question as to whether movements in the long rate induced by changes in the term premium will affect output. Clearly, in the simple analytical framework above, shifts in $\Psi_{n,t}$ would not affect demand. However, in a more elaborate model that recognizes the adjustment costs and partial irreversibility of purchases of certain durable goods, such as residential structures and capital, a change in $\Psi_{n,t}$ may well feed through to the relevant cost of credit that private borrowers face.[11] Still, this second channel has not been tightly formulated in the literature, and we do not employ it.

In summary, long-rate rules appear to have at least the basic prerequisites to be successful in the sense that the central bank appears to have sufficient control over the long rate in order to implement these rules and that manipulation of the long rate appears to provide some measure of control over the economy. The most well-established channel for this two-step linkage of control, and notably the one championed by Reifschneider and Williams (2000) and Eggertsson and Woodford (2003a,b), involves managing expectations of the future path of short rates. In contrast, the cause and effect of changes in term premiums are much less understood.

[10]A determined Fed was able to set yields on long-term U.S. Treasury securities from 1942 to 1951 by standing ready to buy and sell them at a given price. More recently, Bernanke et al. (2004) examine particular episodes of news about the relative supply of Treasury securities, and they conclude that large purchases of long bonds probably would affect long yields through changes in term premiums.

[11]For a discussion incorporating investment into the Euler equation, see Casares and McCallum (2000). For empirical evidence on this issue, see Fuhrer and Rudebusch (2004).

In our analysis then, we focus exclusively on the short-rate expectational channel for the operation of a long-rate policy rule, and we defer issues related to the term premium to future work.

3.2. Conditions for determinacy

We now explore the properties of long-rate rules more formally. We assume that monetary policy is described by a modified version of the Taylor rule in which a particular maturity m-period long bond rate is determined by the current inflation rate

$$i_{m,t} = \theta_m \pi_t. \tag{13}$$

We assume that other bond rates for $n = 2, \ldots, N$ $(n \neq m)$ and the short-term interest rate are all determined implicitly by the expectations theory of the term structure given by (10) with $\Psi_{n,t} = 0$. Also, as discussed in the next subsection, we abstract from the zero bound.

We start by focusing on the 2-period bond rate as the policy rate, which is a simple case that allows us to obtain analytical results; later, we examine longer-maturity policy rates numerically. From the rational expectations hypothesis and after substituting in for the policy rule, we have the equation for the short-term interest rate:

$$i_t = \theta_2(1+\delta)\pi_t - \delta \mathrm{E}_t i_{t+1}. \tag{14}$$

This equation indicates the central change in the model when the long rate is used as the policy instrument: the short rate depends explicitly on expectations of future short rates. Although the 2-period bond rate is set by the central bank as a function of the current inflation rate, no restriction pins down the current short rate; in fact, a continuum of paths for the short rate are consistent with the 2-period bond rate policy, and thus this model may be determinate or indeterminate depending on the eigenvalues of the system.

Finally, we now close the model given by Eqs. (1) and (2) with the long-rate policy rule, which increases the dimension of the reduced form. We have

$$\begin{pmatrix} 1 & 0 & \beta \\ -\alpha & 1 & 0 \\ 0 & \theta_2 & -\frac{1}{1+\delta} \end{pmatrix} \begin{pmatrix} y_t \\ \pi_t \\ i_t \end{pmatrix} = \begin{pmatrix} 1 & \beta & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{\delta}{1+\delta} \end{pmatrix} \begin{pmatrix} \mathrm{E}_t y_{t+1} \\ \mathrm{E}_t \pi_{t+1} \\ \mathrm{E}_t i_{t+1} \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} e_t \\ u_t \end{pmatrix},$$

or, via the appropriate identifications,

$$H x_t = F \mathrm{E}_t x_{t+1} + J \varepsilon_t. \tag{15}$$

To analyze determinacy, write ξ_t as the three-dimensional forecast error (the model has three free variables), and use (15) to write

$$\begin{pmatrix} x_t \\ \varepsilon_t \end{pmatrix} = \begin{pmatrix} F^{-1}H & -F^{-1}J \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_{t-1} \\ \varepsilon_{t-1} \end{pmatrix} + \begin{pmatrix} \xi_t \\ \varepsilon_t \end{pmatrix}. \tag{16}$$

This model is determinate provided the three eigenvalues of $F^{-1}H$ are outside the unit circle.

We say a model exhibits "order M indeterminacy" if there are at least M free variables and if M of the relevant eigenvalues are inside the unit circle so that there are M degrees of expectational freedom (see Evans and McGough, 2003 for details). For the forward-looking IS-AS model, in the cases of determinacy or indeterminacy, there is a unique MSV solution[12] that may be obtained as follows: choose the forecast errors ξ_t so that $z_{it} = 0$ for $i = 1, 2, 3$. Then we may write

$$x_t = \begin{pmatrix} S^{11} & S^{12} & S^{13} \\ S^{21} & S^{22} & S^{23} \\ S^{31} & S^{32} & S^{33} \end{pmatrix}^{-1} \begin{pmatrix} S^{14} & S^{15} \\ S^{24} & S^{25} \\ S^{34} & S^{35} \end{pmatrix} \varepsilon_t. \tag{17}$$

It is straightforward to show that this REE yields the same stochastic processes for y, π, and i as does the model when closed with the short-rate policy $i_t = \gamma\pi_t$, provided the short model is determinate.[13]

Note that because the characteristic polynomial is a cubic, one can take advantage of the formulas for the roots of a cubic equation to analyze the conditions for determinacy. Thus, for the case of a 2-period bond rate, we have the following analytical result.

Proposition 3. *The model is determinate if* $\theta_2 > 1$ *and is indeterminate otherwise.*

This proposition holds regardless of the values of the model parameters, α, β, and δ.

Proposition 3 implies that certain determinate REE that can be obtained when the policy instrument is the 1-period rate cannot be obtained using the stipulated 2-period rate policy rule. In particular, the REE associated with values of $1 < \gamma \leqslant 1 + \delta$ are not obtainable as determinate outcomes from a 2-period bond rate policy rule of this form. (In an REE, the condition that $\theta_2 = \gamma/(1 + \delta)$ holds, while determinacy for a 2-period bond rate policy implies that $\theta_2 = \gamma/(1 + \delta) > 1$.) Below, we examine the effectiveness of alternative specifications of the bond rate policy rule at overcoming this shortcoming of bond rate policies.

Given the prevalence of indeterminacy with long-rate policy rules in the model, the next question is whether there is a natural criterion for selecting some of the equilibria over others. In particular, is the MSV equilibrium associated with the short-rate policy selected by such a criterion? We address these questions now by appealing to the principle of the learnability of the equilibrium.

[12]Following Evans and Honkapohja (2001), we define an MSV solution as one "…which depends linearly on a set of variables, and such that there does not exist a solution which depends linearly on a smaller set of variables." McCallum (1983, 1999a) introduced the notion of an MSV solution. McCallum (2002) has also advocated an additional selection criterion for an MSV solution that is closely connected to E-stability.

[13]In the case of order one indeterminacy, sunspot solutions also exist.

The analysis of stability under learning proceeds exactly as it did above, except now the vector x_t is three dimensional. We suppress the details here. In the case of the 2-period bond rate, determinacy implies stability, as stated in the following proposition.

Proposition 4. *If the model is determinate under the* 2*-period bond rate policy, then the unique REE is stable under learning.*

It is worth noting that it is not in general the case that indeterminacy necessarily implies instability. To see this, consider the case of order one indeterminacy. There is precisely one eigenvalue of $F^{-1}H$ inside the unit circle, and so it must be real. This implies that $H^{-1}F$ has exactly one eigenvalue of norm larger than one, and it is real. Thus the MSV solution is unstable if this eigenvalue is larger than one and stable if this eigenvalue is smaller than negative one. However, for the model at hand, we find numerically that when indeterminacy is introduced, the MSV solution corresponding to the unique equilibrium under the short-rate policy is not stable under learning.

The same condition for determinacy applies when the policy instrument is the rate on a bond whose term is three periods or longer. Although analytical results are not obtainable in these cases, we find numerically that $\theta_m > 1$ is the necessary and sufficient condition for determinacy for values of m between 3 and 8. Again, this condition is invariant to the other model parameters. In addition, as m increases, the set of obtainable determinate REE shrinks. In particular, for an m-period bond rate policy, the determinate REE associated with $\gamma \leqslant \sum_{j=0}^{m-1} \delta^j$ are not obtainable.

3.3. Discussion

The above analysis shows that a central bank that follows a long-rate policy rule of the form $i_{m,t} = \theta_m \pi_t$ may not be able to obtain a first-best outcome that is achievable through a short-rate rule of the form $i_t = \gamma \pi_t$. There are two issues to discuss in this regard.

First, as noted above, we have ignored the zero bound constraint on all nominal rates, which may seem odd since this is a key motivation for using a long-rate instrument in the first place. However, our implicit view is that the steady state level of the nominal rate (taking into account the equilibrium real rate and the inflation target) is sufficiently high that the central bank can move out the yield curve far enough so that at some point future expected short-term interest rates turn positive and the zero bound does not bind on the long-rate instrument. Still, ignoring the zero bound on the *short* rate under a *long* rate policy rule also has an intimate connection with indeterminacy, because, in general, the set of multiple equilibria will be smaller when the zero bound is enforced. Intuitively, the zero bound constraint eliminates some of the possible paths of the short rate that might support a given long rate. For example, if the 2-period rate is at zero, there is only one possible expected path for the current and the next period's short rate under the constraint that the short rate cannot be negative. More generally, however, it will not be the case that the choice of a long rate will imply zero short rates throughout the term of the bond, and in such cases, the issue of multiplicity may arise.[14]

Second, the existence of multiple equilibria using a long-rate instrument raises questions of whether the central bank can choose among them. As noted in Section 3.1, if the central bank can signal a particular path for the short rate through its statements or actions, it may be able to center expectations and obtain a particular equilibrium. Of course, altering the public's perception of how policy will be conducted once in a while in an ad hoc fashion is likely to be difficult. That is, a central bank that typically follows a short-rate rule but switches to some type of signaling or communications strategy at the zero bound may find some hard sledding in trying to convey its commitment credibly. A long-rate policy rule, which could be followed and communicated at all times and in all situations, would seem to have some advantage over more occasional strategies.

4. Backward-looking policy rules

The preceding section highlights the problem of indeterminacy in a standard New Keynesian model, where monetary policy is described by a rule in which the bond rate is determined by the current inflation rate. However, as stressed by McCallum (1999b), policymakers observe data with a lag that reflects the delays inherent in collecting and compiling macroeconomic data. Thus, according to this view, the policies studied in Sections 2 and 3 are not operational in practice. In this section, we analyze alternative specifications of both the short-rate rule and the long-rate rule that satisfy McCallum's operational criterion, namely, that policy responds only to information available to the policymaker at the time of the decision. In particular, we assume that the current-quarter setting of policy is determined by the lagged inflation rate.

In addition to making the policy rule operational, this modification implies that lagged inflation is a state variable in the system, which has important implications for the determinacy and stability characteristics of the model. As noted above, in the benchmark New Keynesian model, the only state variables are the shocks; therefore, if policy responds only to the contemporaneous inflation rate and if the shocks are iid, then inflation and the output gap display no dynamics, and there exists a single MSV solution. However, with the operational policy rule, the system is dynamic and, as a result, the nature of indeterminacy potentially generated by long-rate policies changes. In particular, such policies can give rise to multiple MSV solutions, some or all of which may be unstable under learning.

In the following, we abstract from the shocks to the two equations and concentrate on the deterministic dynamics of the system, ignoring for the time being the possibility of sunspot equilibria. This simplifies the analysis of the system and facilitates the study of multiple MSV solutions. At the end of this section, we return to the issue of stochastic disturbances and the possibility of sunspot equilibria.

[14]Taking account of the zero bound would require abandoning the linearity assumption that facilitates our analysis. We leave a nonlinear analysis of the long policy instrument to future research.

For notational convenience, define $\psi \equiv \alpha\beta$. Assuming that the short rate is the policy instrument, we may then write the system as

$$\begin{pmatrix} 1 & 0 & \beta\gamma \\ -\alpha & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} y_t \\ \pi_t \\ \pi_{t-1} \end{pmatrix} = \begin{pmatrix} 1 & \beta & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathrm{E}_t y_{t+1} \\ \mathrm{E}_t \pi_{t+1} \\ \pi_t \end{pmatrix}, \tag{18}$$

or, with the appropriate identifications,

$$Hx_t = F\mathrm{E}_t x_{t+1}. \tag{19}$$

Decoupling the dynamics along eigenspaces as before and imposing perfect foresight we obtain the MSV solutions of the form

$$\begin{pmatrix} y_t \\ \pi_t \end{pmatrix} = \hat{A}\pi_{t-1}.$$

Denote the reduced-form coefficient relating the n-period rate to the lagged inflation rate by G_n. Then, the 2-period rate is given by

$$i_{2,t} = G_2\pi_{t-1} = \frac{\gamma}{1+\delta}(1 + \delta\hat{A}_{21})\pi_{t-1}.$$

The properties of the model when the short rate is the policy instrument are summarized by the following proposition:

Proposition 5. *Assume* $i_t = \gamma\pi_{t-1}$ *and* $0 < \psi < 1$. *Then*

1. *for* $0 < \gamma \leqslant 1$, *the model is indeterminate*,
2. *for* $1 < \gamma < 1 + 4/\psi$, *the model is determinate*,
3. *for* $1 + 4/\psi \leqslant \gamma$, *the model is explosive*.

The lower bound on γ necessary to achieve a determinate REE is the same as when the short rate is determined by the current inflation rate. Note, however, that when policy depends too strongly on lagged inflation, the model is explosive owing to "instrument instability."

To examine stability under learning, we write the reduced form model as

$$\begin{pmatrix} y_t \\ \pi_t \end{pmatrix} = A \begin{pmatrix} \mathrm{E}_t y_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{pmatrix} + B \begin{pmatrix} y_{t-1} \\ \pi_{t-1} \end{pmatrix}.$$

The PLM consistent with the MSV representations is given by

$$\begin{pmatrix} y_t \\ \pi_t \end{pmatrix} = a + b \begin{pmatrix} y_{t-1} \\ \pi_{t-1} \end{pmatrix},$$

which generates the following T-map:

$$a \longrightarrow A(I_2 + b)a$$
$$b \longrightarrow Ab^2 + B.$$

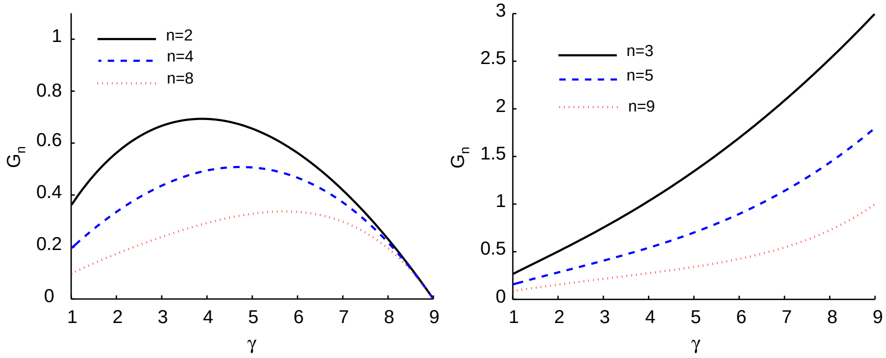

Fig. 1. Sensitivity of long-term bond rates to inflation.

Recall that stability under learning requires that the real part of the eigenvalues of the T-map's derivative be less that one. The relevant Jacobians are given by

$$DT_a = A(I_2 + b)a,$$
$$DT_b = b' \otimes A + I_2 \otimes Ab.$$

Numerical results indicate that for $0 < \gamma \leqslant 1$, that is, if the model is indeterminate, then all MSV representations are unstable under learning, and for $1 < \gamma < 4/\psi + 1$, the unique REE is stable under learning.[15]

Assuming that monetary policy induces a determinate REE, the reduced-form relationship between the bond rate and the lagged inflation rate is uniquely determined; however, the mapping back from a particular value of G_n to γ in general is not. Fig. 1 plots the reduced-form coefficient of bond rates of various maturities on lagged inflation implied by different values of γ. (For the purposes of this figure, we have assumed that $\alpha = 1, \beta = 0.5$, and $\delta = 1$.) Each value of γ implies a unique value of G_n. But, for this parameterization of the model, for even values of n, each value of G_n can be supported by two real values of γ, while for odd values of n, each value of G_n is supported by one real value of γ. This nonuniqueness reflects the fact that G_n is a polynomial of degree n in γ.[16]

The intuition for the finding that a single bond rate reduced-form relationship to inflation can be consistent with more than one short-rate policy rule is illustrated by considering two short-rate policy rules, one that reacts strongly to lagged inflation, say with $\gamma = 7$, and one that responds relatively timidly, say with $\gamma = 1.2$. A short-rate policy that responds strongly to inflation implies that inflation will sharply overshoot its target in the next period. In that case, the reduced-form response of the 2-period bond rate to inflation will be modest, reflecting the strong initial response and the reversal of interest rates in the next period. A policy rule that responds less

[15]It can be shown analytically that the eigenvalues of the Jacobians depend on ψ, but not on α and β independently. Interestingly, the MSV coefficient $\hat{A}$ does depend on α and β independently.

[16]Note that in general there will also exist complex roots to this polynomial in γ, but these equilibria will not be MSV solutions and thus will not be of the form $i_t = \gamma\pi_{t-1}$.

aggressively to inflation will feature a smaller movement in interest rates this period and less of a reversal in the following period, yielding the same relationship between the bond rate and inflation of about 0.4, as shown in the left panel of Fig. 1.

We now analyze policies in which the 2-period rate is the policy instrument, determined by the lagged inflation rate:

$$i_{2,t} = \theta_2 \pi_{t-1}.$$

For this rule to be consistent with the unique equilibrium implemented by a short-rate policy, we must have

$$\theta_2 = \frac{\gamma}{1+\delta}(1 + \delta \hat{A}_{21}).$$

Let $\chi = (1+\delta)\theta_2$. The model may then be written

$$\begin{pmatrix} 1 & 0 & \beta & 0 \\ -\alpha & 1 & 0 & 0 \\ 0 & 0 & 1 & -\chi \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_t \\ \pi_t \\ i_t \\ \pi_{t-1} \end{pmatrix} = \begin{pmatrix} 1 & \beta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\delta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \mathrm{E}_t y_{t+1} \\ \mathrm{E}_t \pi_{t+1} \\ \mathrm{E}_t i_{t+1} \\ \pi_t \end{pmatrix},$$

or, with the appropriate identifications,

$$H\hat{x}_t = F\mathrm{E}_t \hat{x}_{t+1}.$$

Whether the model is determinate depends on the eigenvalues of the matrix $F^{-1}H$.

If the model is indeterminate, there may be multiple MSV solutions and each MSV will be consistent with a distinct short-rate policy rule. Order one indeterminacy implies the existence of possibly two MSV solutions, while order two indeterminacy implies the existence of possibly three MSV solutions, and so on.[17] We note that for calibrations corresponding to Figs. 2 and 3, all relevant eigenvalues are real, so that in case of indeterminacy, multiple MSV solutions exist. Stability under learning is as above, and we suppress the details. Analytical results regarding determinacy and stability are not tractable; instead, we present numerical results in what follows.

Bond rate policies that respond to lagged inflation are prone to generating multiple MSV solutions, where each MSV corresponds to a different set of coefficients describing the reduced form of the solution. Fig. 2 plots the determinacy and stability characteristics of the equilibria resulting from a 2-period rate policy associated with combinations of ψ and γ. In the lower right region, the parameter combinations yield a determinate REE that is also stable under learning; the remaining regions are all characterized by indeterminacy. For some parameter constellations yielding two MSV solutions and labeled "stable" in the chart, only one MSV is stable under learning, and, thus, a stability criterion may be used to select that MSV. However, in other regions, labeled "unstable," stability cannot be applied as a selection criterion because all MSVs are unstable. In such cases, one cannot determine a priori which equilibrium would obtain without applying some additional

[17] The number of MSV solutions will depend on the number of real eigenvalues.

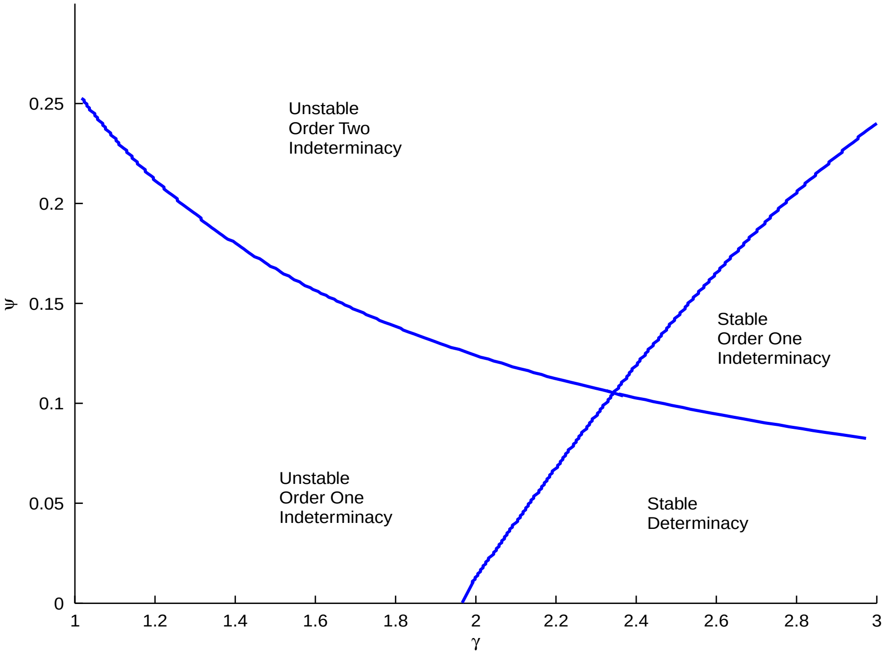

Fig. 2. Determinacy and stability for 2-period bond rate policy.

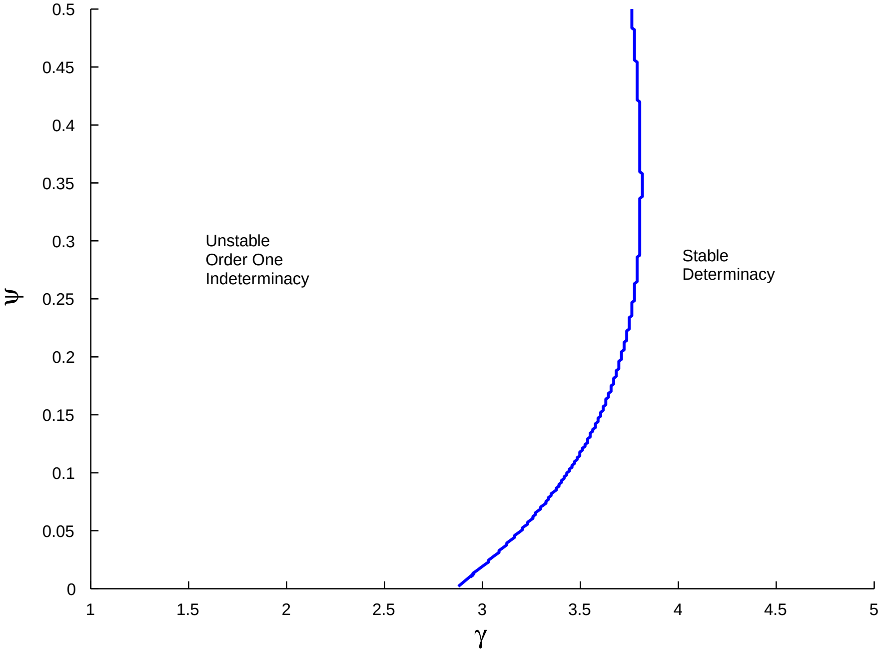

Fig. 3. Determinacy and stability for 3-period bond rate policy.

restrictions on the model.[18] Importantly, these MSVs differ in the behavior of all endogenous variables and thus have first-order effects on the welfare of the representative household.

The problem of multiple MSV solutions can be even more acute with longer maturity policy instruments. Fig. 3 shows the determinacy and stability characteristics of the equilibria resulting from a 3-period bond rate policy; for a range of values of γ and ψ, the 3-period bond rate policies yield indeterminacy and instability, and only when policy is very responsive to lagged inflation does a unique stable REE obtain. For a wide range of intuitively "reasonable" values of γ, there exist two MSV representations that are both unstable under learning.

So far, we have abstracted from sunspot equilibria in this discussion. In fact, one can show that the bond rate policies that respond to lagged inflation and that yield multiple MSV solutions also allow sunspot equilibria of the type analyzed in the previous section.

A key finding in this section is that policies in which the bond rate is the policy instrument not only can lead to sunspot equilibria, but also can generate multiple MSV representations when lagged inflation is a state variable. It is worth emphasizing that this conclusion also applies to a wide range of macro models that incorporate forms of inertia in output and inflation, as studied in Woodford (2003). Thus, based on this analysis, such policies appear to be problematic.

5. Forward-looking long-rate policy rules

Given the prevalence of indeterminacy associated with long-rate policy rules that respond to contemporaneous or lagged inflation, we now turn to alternative specifications of long-rate policies that are less susceptible to indeterminacy. We show that properly specified forward-looking long-rate policy rules are immune to the indeterminacy and instability problems plaguing the long-rate rules studied in the previous two sections. In this section, we focus on policies that respond to contemporaneous and expected future inflation.[19]

A basic problem with long-rate policies that respond only to contemporaneous inflation is that they rely on reduced-form behavior that may be violated in the economy. Consider, for example a 2-period bond rate policy (12) with coefficient $\theta_2 = 1$, corresponding, in the absence of sunspots, to a short-rate policy with $\gamma = 2$, assuming, for the moment, that $\delta = 1$. But, a sunspot that raises expected inflation and thereby current inflation by one percentage point calls forth a one percentage point increase in the bond rate, which supports the higher inflation rate. One solution to this problem is to impose $\theta_2 > 1$, which assures a determinate stable REE. However, such a policy can only achieve a strict *subset* of REE obtainable when the

[18] Recall that, according to McCallum's definition, there is always a unique MSV solution, and his MSV solution corresponds to one of the two we consider.

[19] Preliminary results suggest that the findings for operational forward-looking rules are the same as for the rules that we study here.

short rate is the policy instrument, where this subset is the REE associated with values of $\gamma > 2$. Thus, long-rate policies may be inefficient if the optimal choice of $\gamma \in (1, 2)$.

A forward-looking long-rate policy rule responds directly to sunspots and other forms of "off-equilibrium" behavior that affect expectations and in so doing is able to deliver the desired determinate and stable REE. Consider the 2-period long-rate policy rule of the form

$$i_{2,t} = \frac{\theta_2}{1+\phi}(\pi_t + \phi \mathrm{E}_t \pi_{t+1}), \; 0 \leqslant \phi \leqslant \delta, \tag{20}$$

where θ_2 is the response coefficient to the weighted average of current and expected future inflation rates. The case of $\phi = 0$ was analyzed in Section 3. If $\phi = \delta$, then we say that the "duration" of the inflation forecast appearing in the policy rule matches that of the bond. Now, consider the same thought experiment of a sunspot affecting inflation expectations described above, but assume $\phi = \delta < 1$. Note that under the assumption of $\phi = \delta$, the short-rate policy described by γ yields an equilibrium in which the long-rate policy equation holds exactly with $\theta_2 = \gamma$. So, under the long-rate policy corresponding to $\gamma = 2$, a one percentage point rise in inflation and inflation expectations raises the 2-period bond rate by two percentage points, which implies a rise in average real short-term interest rates, which stabilizes the economy.

For the case of forward-looking policies that use the 2-period bond rate as the policy instrument, we obtain analytical results regarding determinacy and stability.

Proposition 6. *If $\phi \leqslant \delta < 1$, the model is determinate if $\theta_2 > 1$ and is indeterminate otherwise. If $\phi < \delta = 1$, the model is determinate if $\theta_2 > 1$ and is indeterminate otherwise. If $\phi = \delta = 1$, the model is indeterminate. Finally, if the model is determinate then the unique equilibrium is stable under learning.*

These conditions are invariant to the values of the parameters α and β. We have also considered forward-looking longer maturity long-rate policies of the form

$$i_{m,t} = \frac{\theta_m}{(\sum_{j=0}^{m-1} \phi^j)} \mathrm{E}_t \sum_{j=0}^{m-1} \phi^j \pi_{t+j}, \quad 0 \leqslant \phi \leqslant \delta. \tag{21}$$

Although we are not able to derive analytical results for the case of $m > 2$, based on numerical investigation, we find the same results as in the case of $m = 2$.

An important implication of this proposition is that by setting ϕ equal to δ (assuming $\delta < 1$), any REE associated with a determinate short-rate policy is obtainable using a long-rate policy and the REE resulting from the long-rate policy is determinate. In the case of $\delta = 1$, any REE associated with a determinate short-rate policy may be obtained using a long-rate policy by simply setting $\phi = 1/\gamma$ and $\theta_2 = (1+\gamma)/2$. Our numerical results indicate the same conclusion applies when longer-duration bonds are used as the policy instrument.

As noted in the introduction, an important implication of these results is that in order to assure determinacy, policy should be explicitly forward-looking when the policy instrument is inherently forward-looking. This contrasts with the findings of

Bernanke and Woodford (1997) and Levin et al. (2003) that a forward-looking monetary policy using a short-term interest rate instrument may well generate indeterminacy, unlike policies that respond to current or lagged inflation.

6. Conclusion

One proposed solution to the zero bound problem is for the central bank to use a longer-term bond rate as the policy instrument. In this paper, we show that such a policy can easily lead to indeterminacy; furthermore, in some cases, more than one of the resulting multiple equilibria may be stable under learning. We show that the problem of indeterminacy can be avoided if policy is explicitly forward-looking. Specifically, a forward-looking policy rule in which the bond rate is determined by expected inflation over the maturity of the bond produces a determinate and stable equilibrium. The bottom line of this stylized analysis is that, although "moving out the yield curve" in response to the zero bound involves more considerations than might be apparent at first, a careful central bank conceivably could pursue an effective long-rate policy. However, it should be stressed that our austere theoretical analysis is silent on many important practical issues. For example, our framework cannot address questions such as the tradeoff in the choice of maturity to use as the policy instrument. Moreover, we have not considered the characteristics of long-rate policy rules that are able to best mimic the optimal equilibrium under the zero bound. Such an analysis requires an empirical model and explicit incorporation of the zero bound on interest rates and is a fruitful direction for future research.

Of course, lower future short nominal rates and lower current and future long nominal rates could accomplish the goal of trying to reduce real interest rates enough in order to stimulate the economy. However, higher expectations of future inflation can also lower real interest rates, and there are many proposals to exploit such expectational policy levers on the economy at the zero bound (e.g., Krugman, 1998; Svensson, 2001, 2003; and McCallum, 2000, 2002).[20] Our analysis appears to provide a cautionary note as well for this general class of proposals that involve expectational policy instruments like the long rate. In particular, trying to manipulate the average expected inflation rate over the next, say, 5 years by means of an inflation target may not be sufficient, even if credible, to determine a single outcome, because there may be many possible paths for expected prices that are consistent with a long-run average inflation rate.

Finally, our analysis of the manipulation of the long rate should not be construed as suggesting that we think this is the only or even the most effective policy that could be pursued at the zero bound when economic stimulus needs to be applied. But we do believe that given the current central bank unanimity regarding the short rate as the instrument of monetary policy, manipulation of the long rate is the most likely

[20] In addition, direct effects on aggregate demand through the exchange rate and monetary base channels may be exploited.

first step in overcoming the zero bound—again, as suggested by Chairman Greenspan's views in the epigraph.

Appendix

This appendix provides the proofs for Propositions 3, 5, and 6; the proofs of Propositions 1 and 2 are found in Woodford (2003) and Evans and Honkapohja (2001), respectively, and the proof of Proposition 4 is identical to that of Proposition 2.

The proofs of Propositions 3, 5, and 6 require analyzing when the roots of a cubic polynomial are of unit modulus. This analysis will be aided by the following lemma.

Lemma. If y and z are roots of the polynomial $x^3 - Ax^2 + Bx - C$, and if $y = \bar{z}$ (where $\bar{z}$ is the conjugate of z), and if $|z| = 1$, then

$$A - C = \frac{B-1}{C}.$$

Proof. Let z_i, $i = 1, 2, 3$, be the roots of the polynomial. Expanding

$$(x - z_1)(x - z_2)(x - z_3)$$

shows

$$A = z_1 + z_2 + z_3, \tag{22}$$

$$B = z_1 z_2 + z_1 z_3 + z_2 z_3, \tag{23}$$

$$C = z_1 z_2 z_3. \tag{24}$$

(We will use these equations repeatedly in the proofs of the propositions.) Without loss of generality, assume $z_1 = y$ and $z_2 = z$. Write $z = a + b\mathrm{i}$. Then $z_1 + z_2 = 2a$ and $z_1 z_2 = 1$. Then by (24), $z_3 = C$, by (23), $2az_3 = B - 1$, and by (22), $A = 2a + z_3$. $\square$

Proof of Proposition 3. To analyze determinacy, we consider the nonstochastic model, and eliminate variables to write down a single dynamic equation in inflation. Letting $\psi = \alpha\beta$, we obtain

$$\delta\pi_{t+3} + (1 - \delta(2+\psi))\pi_{t+2} + (\delta - (2+\psi))\pi_{t+1} + (\theta_2\psi(1+\delta) + 1)\pi_t = 0. \tag{25}$$

The associated cubic, which we write in the form $x^3 - Ax^2 + Bx - C = 0$, has, for coefficients,

$$A = 2 + \psi - \frac{1}{\delta},$$

$$B = 1 - \frac{1}{\delta}(2 + \psi),$$

$$C = -\frac{1}{\delta}\,(\theta_2\psi(1+\delta) + 1).$$

The model is determinate if the roots of this polynomial are all outside the unit circle, and indeterminate otherwise. To compute the regions in parameter space corresponding to determinacy and indeterminacy, we analyze when the roots lie on the unit circle, and then appeal to continuity.

Suppose the cubic has a complex root, $z_1 = a + b\mathrm{i}$, of unit modulus, and further suppose the imaginary component, b, is nonzero. Then its complex conjugate is also a root, and we assume it is given by z_2. Let $\hat{A} = \theta_2\psi(1+\delta)$. By (24), $z_3 = -(1/\delta)(\hat{A}+1)$. This, combined with (23), implies $2a = (2+\psi)/(\hat{A}+1)$. By (22), $2a = 2 + \psi + (1/\delta)\hat{A}$. Setting these two equations equal to each other implies

$$2+\psi = (1+\hat{A})(2+\psi) + \frac{1}{\delta}\hat{A}(\hat{A}+1).$$

Dividing by $2+\psi$ and simplifying results in $\hat{A}<0$, which contradicts our assumptions on the model's parameters. We conclude that $b = 0$.

Now assume $z_1 = 1$. Then by (22), $z_2 + z_3 = 1 + \psi - 1/\delta$. Combining this with (23) implies $z_2z_3 = -(1/\delta)(\psi(1+\delta)+1)$, and (24) implies $z_2z_3 = -(1/\delta)(\hat{A}+1)$. Setting these last two equations equal to each other yields $\theta_2 = 1$.

Finally, assume $z_1 = -1$. Then by (22), $z_2 + z_3 = 3 + \psi - (1/\delta)$, and combining this with (23) yields

$$z_2z_3 = 4 + \frac{1}{\delta}(\psi(\delta-1)-3).$$

And, (24) gives $z_2z_3 = (1/\delta)(\hat{A}+1)$. Combining these last two equations and solving for θ, we have

$$\theta_2 = \frac{4(\delta-1)-\psi(1-\delta)}{\psi(1+\delta)},$$

which contradicts the assumption that $\theta_2 > 0$.

We have shown that if the roots of the polynomial cross the unit circle, then our parameter restrictions imply that the crossing must occur at $z = 1$, and the associated value of θ_2 must be one. By specifying a calibration of the model, and choosing $\theta_2 < 1$, we find numerically that the model is indeterminate, and choosing $\theta_2 > 1$, we find the model is determinate. The result then follows from the fact that the roots of the polynomial are continuous functions of the parameters. □

The proofs of Propositions 5 and 6, below, are quite similar to that of Proposition 3 and thus our exposition will be more brief.

Proof of Proposition 5. Writing down the associated polynomial, Eqs. (22)–(24) yield

$$2+\psi = z_1 + z_2 + z_3, \tag{26}$$

$$1 = z_1z_2 + z_1z_3 + z_2z_3, \tag{27}$$

$$-\psi\gamma = z_1z_2z_3. \tag{28}$$

First assume $z_1 = a + bi$ has unit norm and $b \neq 0$. By (28), $z_3 = -\psi\gamma < 0$, so that by (27), $a = 0$. Then by (26), $z_3 = 2 + \psi > 0$, which contradicts $z_3 = -\psi\gamma < 0$. Thus $b = 0$.

Now suppose $z_1 = 1$. Then by (26), $z_2 + z_3 = 1 + \psi$, which, when combined with (27), yields $z_2 z_3 = -\psi$. Also, (28) implies $z_2 z_3 = -\psi\gamma$. These last two equations combine to yield $\gamma = 1$.

Finally, suppose $z_1 = -1$. Then by (26), $z_2 + z_3 = 3 + \psi$, which, when combined with (27), yields $z_2 z_3 = 4 + \psi$. Also, (28) implies $z_2 z_3 = \psi\gamma$. These last two equations combine to yield $\gamma = 1 + 4/\psi$.

Again, we may finish the proof by specifying a calibration, choosing values of γ within the regions set off above, numerically computing roots, and then appealing to continuity. $\square$

Proof of Proposition 6. Let $\hat{A} = \theta_2(1+\delta)/(1+\phi)$. The parameters of the associated polynomial are given by

$$A = 2 + \psi - \frac{1}{\delta},$$
$$B = \frac{\hat{A}\phi + \delta - \psi - 2}{\delta},$$
$$C = -\frac{1}{\delta}(\hat{A} + 1).$$

First assume $z_1 = a + b\mathrm{i}$ has unit norm and $b \neq 0$. The software Mathematica may be used with the Lemma to solve for $\hat{A}$: we find that $\hat{A} = 0$ or

$$\hat{A} = -(1 + \delta(2 + \phi + \psi)) < 0,$$

which contradicts the restrictions on our parameters.

Now assume $z_1 = 1$. Then by (22), $z_2 + z_3 = 1 + \psi - 1/\delta$, and combining this with (23) yields $z_2 z_3 = B - (1 + \psi - 1/\delta)$. And, (24) gives $z_2 z_3 = -(1/\delta)(\hat{A} + 1)$. Setting these last two equations equal to each other yields $\theta_2 = 1$.

Now assume $z_1 = -1$ and $\phi < 1$. Then by (22), $z_2 + z_3 = 3 + \psi - 1/\delta$, and combining this with (23) yields $z_2 z_3 = B + (3 + \psi - 1/\delta)$. And, (24) gives $z_2 z_3 = (1/\delta)(\hat{A} + 1)$. Setting these last two equations equal to each other, and using Mathematica to solve for θ_2 yields

$$\theta_2 = \frac{(1-\delta)(1+\phi)(4+\psi)}{\psi(1+\delta)(\phi - 1)} < 0,$$

which contradicts the restrictions on our parameters.

Finally, if $\phi = 1$ and $\delta = 1$, then there is a root of the polynomial equal to -1, as can be shown using Mathematica.

Proceeding as in the proofs of Propositions 3 and 5 completes the proof. $\square$

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