The Lucas critique revisited
Assessing the stability of empirical Euler equations for investment

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Abstract

Lucas (1976) argued that the parameters of traditional macroeconometric models depended crucially on agents' expectations and were unlikely to remain stable in a changing economic environment. In response, econometric modeling has focused on the estimation of rational expectations models that have an explicit structural interpretation - Euler equations in particular. Thus, a natural, though little acknowledged, criterion for judging the success of empirical Euler equations is the stability of their 'deep', structural parameters. Examining split-sample tests over multiple breakpoints as well as the sequence of subsample model estimates, we find considerable instability in the estimated parameters of a standard Euler equation of investment.

Key words: Lucas critique; Euler equation; Business investment

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1. Introduction

In 1976, Lucas issued a trenchant critique of traditional macroeconometric models for their failure to account explicitly for agents' expectations of future variables. He argued that the coefficients of the behavioral equations of these models depended, in part, on the parameters describing the formation of agents' expectations; furthermore, under rational expectations, the expectations parameters will reflect agents' understanding of the underlying economic structure. Accordingly, if there were a structural change in the laws of motion for the exogenous variables, the coefficients of the models' behavioral equations could not be expected to remain stable. As a prominent example, Lucas criticized Hall and Jorgenson's (1967) use of a neoclassical model of equipment investment to gauge the effect of a change in the investment tax credit because such a policy intervention should change individuals' expectations and thus their decision rules.

The basic thrust of the Lucas critique—that the coefficients of reduced-form models are not invariant to structural changes—had been acknowledged well before 1976.1 However, Lucas' version of this critique, which stressed the crucial role of expectations, was widely viewed as a devastating indictment of traditional consumption, wage–price, and investment equations. Following the Lucas critique, the coefficients of such empirical models were considered to be reduced-form, 'shallow' parameters. This resulted in a major reorientation of the theory and practice of econometric modelling. As stated by Hansen and Sargent (1980, pp. 7–8):2

The implication of Lucas's observation is that instead of estimating the parameters of decision rules, what should be estimated are the parameters of agents' objective functions and of the random processes that they faced historically. Disentangling the parameters governing the stochastic processes that agents face from the parameters of their objective functions would enable the econometrician to predict how agents' decision rules would change across alterations in their stochastic environment. Accomplishing this task is an absolute prerequisite of reliable econometric policy evaluation.

The subsequent research program of rational expectations econometrics has attempted to estimate the underlying parameters of taste and technology governing objective functions. In particular, much research has focused on

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1 Lucas himself points out that Marschak (1953) and Tinbergen (1956) raised similar criticisms. See Favero and Hendry (1992) for a comprehensive review and interpretation of the Lucas critique.

2 Also see Sargent (1982) and Wallis (1980).
the estimation of the stochastic first-order conditions for optimal choice by a rational, forward-looking representative agent. Indeed, following Hall (1978), this 'Euler equation' modelling strategy has dominated empirical work in consumption. Euler equations also have been widely estimated for investment spending, especially in the past few years. A list of empirical Euler equations for investment includes: Abel (1980), Pindyck and Rotemberg (1983), Shapiro (1986a,b), Gilchrist (1990), Himmelberg (1990), Bizer and Sichel (1991), Gertler, Hubbard, and Kashyap (1991), Hubbard and Kashyap (1992), Whited (1992), Ng and Schaller (1992), and Carpenter (1992). These papers estimate the first-order conditions of the firm's intertemporal optimization problem given production and adjustment cost functions, and they generally view the resulting estimates as shedding light on 'deep' technological parameters.

Although Euler equations have become a very popular approach to modelling investment, there has been surprisingly little examination of their empirical adequacy. Without exception, the papers cited above judge model adequacy using only Hansen's (1982) $J$-test, which tests the orthogonality of instruments and errors. Unfortunately, the $J$-test is not a good test of overall model specification. Newey (1985) analyzed the power of Hansen's test and found that the test could not distinguish a variety of local alternatives to the Euler equation. Following Newey's analysis, Ghysels and Hall (1990a) provided a particularly apt illustration of the inadequacy of the $J$-test: they showed that the test had asymptotic power equal to size against a class of local alternatives characterized by parameter drift. As Ghysels and Hall (1990a,b) argue, such alternatives are precisely the relevant ones to confront when testing the success of estimated Euler equations. Because the genesis of the Euler equation approach can be found in the charge that the parameters of traditional models were unstable, it is appropriate that Euler equations be judged on their ability to deliver stable estimates of the deep parameters of taste and technology.

In this paper, we examine an investment Euler equation that is typical of those previously estimated in the literature. However, rather than relying on the $J$-test as the sole model diagnostic, we apply a battery of structural stability tests. In large part, our examination is inspired by Ghysels and Hall (1990a,b), who stress that structural stability tests are a natural diagnostic for the Euler equation model. We analyze the stability of the Euler equation with two basic techniques. Our first set of tests splits the sample into two parts and compares the Euler equation estimates across the two subsamples. These split-sample tests for structural stability generalize the usual $F$-test for structural change in a linear regression discussed by Chow (1960). Second, we consider the sequence of model estimates obtained by starting with a small set of observations and progressively enlarging the estimation sample one observation at a time. This sequence of subsample estimates provides direct evidence concerning parameter drift. Both types of tests indicate that the standard investment Euler equation exhibits substantial parameter instability.
This paper is organized as follows. Section 2 derives the Euler equation that we shall evaluate and presents the full-sample estimates of its parameters. Section 3 conducts formal split-sample tests for parameter stability using a range of sample breakpoints. Section 4 presents the sequences of subsample estimates of the model’s parameters. Section 5 concludes.

2. An investment Euler equation

2.1. The investment model

The model studied in this paper is representative of investment Euler equations that have been estimated in the literature. Although the assumptions underlying these models are open to criticism, we have adopted these assumptions because we wished to assess the ability of existing models to meet the Lucas critique. The key features of our model are as follows:

- The firm’s production function is assumed to be Cobb–Douglas with constant returns to scale. Specifically,

\[ Y_t = F(K_{t-1}, L_t) = AK_{t-1}^{\gamma}L_t^{(1-\gamma)}, \]

where \( Y_t \) and \( L_t \) are output and employment during period \( t \) and \( K_{t-1} \) is the capital stock at the end of period \( t-1 \).

- Both capital and labor are quasi-fixed factors subject to quadratic adjustment costs. Let \( AL_t = L_t - L_{t-1} \) denote the change in employment, and let \( I_t \) denote gross investment during period \( t \). Then, the specification of adjustment costs is

\[ C(I_t, K_{t-1}, AL_t, L_{t-1}) = [\phi_0 IK_t + \phi_1/2 IK_t^2] K_{t-1} \]

\[ + [\phi_0 (AL_t/L_{t-1}) + \phi_1/2 (AL_t/L_{t-1})^2] L_{t-1} \]

\[ + \gamma (AL_t/L_{t-1}) (IK_t) K_{t-1}, \]

where \( IK_t = I_t/K_{t-1} \). Note that the cost of adjusting capital input is allowed (with \( \gamma \neq 0 \)) to depend on simultaneous adjustments in employment.

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3Thus, capital purchased during \( t - 1 \) does not become productive until period \( t \). Such a lag is required by our data, which measure investment at the time a capital good is shipped from the manufacturer, rather than the later date when it is put into service.

4 Eq. (2) is characterized by constant returns to scale. With constant returns in both the production and adjustment cost functions, the firm’s optimal scale is undetermined; rather, the resulting Euler equation implies a structural relation between \( IK \) and other variables.
The partial derivatives of \( C \) with respect to \( I_t \) and \( K_{t-1} \), needed to specify the investment Euler equation, are

\[
C_{I_t} = a_0 + a_1 I_t + \gamma \left( \Delta L_t / L_{t-1} \right),
\]

\( \text{(3)} \)

\[
C_{K_{t-1}} = -(a_1/2) I_{t}^2.
\]

(4)

For the firm's investment decision to be well-defined, marginal adjustment costs must be increasing with the level of investment. That is, \( \partial^2 C / \partial I_{t}^2 = a_1 / K_{t-1} \) must be greater than zero, which implies that \( a_1 > 0 \). We have no strong priors on the signs of \( a_0 \) or \( \gamma \).

- All markets are assumed to be perfectly competitive, implying that the price of output, the price of capital goods, and the wage rate are exogenous. We normalize both of the input prices by the price of output \( (p_t) \), and denote the resulting real price of capital goods and real wage by \( p^I_t \) and \( w_t \), respectively.

- The firm's discount rate is assumed to be exogenous, so financing decisions are irrelevant to the optimal investment path. We denote the time-varying discount rate used by the firm by \( r_t \) and the corresponding discount factor by \( \beta_t = 1/(1 + r_t) \).

- The depreciation rate on the firm's capital stock is assumed to be a constant \( \delta \).

Following the usual practice in the literature, we assume that the firm maximizes the expected present value of real future profits,

\[
V_t = E_t \left[ \sum_{s=t}^{\infty} \beta^*_{t,s} R_s \right],
\]

(5)

where

\[
\beta^*_{t,s} = \prod_{j=t+1}^{s} \beta_j
\]

is the multi-period discount factor (with \( \beta^*_{t,t} = 1 \)) and

\[
R_s = F(K_{s-1}, L_s) - C(I_s, K_{s-1}, \Delta L_s, L_{s-1}) - w_s L_s - p_s^I I_s
\]

is real profit in period \( s \). Firms maximize (5) by choosing \( I_s, K_s, \) and \( L_s \) for all \( s \geq t \), subject to the usual constraint on the evolution of their capital stock:

\[
K_t = (1 - \delta) K_{t-1} + I_t.
\]

(6)

To carry out this constrained optimization, we define the Lagrangian

\[
L_t = E_t \left[ \sum_{s=t}^{\infty} \beta^*_{t,s} [R_s - \lambda_s (K_s - (1 - \delta) K_{s-1} - I_s)] \right].
\]
Setting $\partial L_i/\partial x_s = 0$, with $x_s = (I_s, K_s, L_s)$, yields the first-order conditions linking variables dated $s - 1, s$, and $s + 1$. In particular, for $s = t$,

$$l: \quad p_t^l + C_t = \lambda_t, \quad (7)$$

$$K: \quad E_t \beta_{t+1}(F_K - C_K) = \lambda_t - (1 - \delta)E_t \beta_{t+1} \lambda_{t+1}, \quad (8)$$

$$L: \quad F_L - C_L + E_t \beta_{t+1}[C_{AL_{t+1}} - C_L] = w_t. \quad (9)$$

Eq. (7) is the first-order condition for investment. At the optimal level of investment spending, the full cost of acquiring and installing a unit of the capital good must equal $\lambda_t$, the shadow value of a marginal increase in the capital stock at time $t$. Eq. (8), the first-order condition for capital, equates the net marginal return on capital in period $t + 1 (F_K - C_K)$ to the user cost of capital. Eq. (9) equates the marginal product of labor, net of adjustment costs, to the real wage.

To derive the investment Euler equation, combine Eqs. (7) and (8) to eliminate $\lambda_t$ and $\lambda_{t+1}$, yielding

$$E_t \beta_{t+1}(F_K - C_K) - p_t^l - C_t + E_t \beta_{t+1}(1 - \delta)[p_{t+1}^l + C_{L_{t+1}}] = 0. \quad (10)$$

Given the production function specified in Eq. (1), the marginal product of $K_t$ equals $\theta Y_{t+1}/K_t$. Substituting this expression for $F_K$ into (10), along with the expressions for the partial derivatives of $C$ from (3) and (4), we obtain

$$E_t \beta_{t+1}[\theta Y_{t+1}/K_t + (\alpha_1/2)IK^{2}_{t+1}] - p_t^l - \alpha_0 - \alpha_1 IK_t - \gamma(\Delta L_t/L_{t-1}) + E_t \beta_{t+1}(1 - \delta)[p_{t+1}^l + \alpha_0 + \alpha_1 IK_{t+1} + \gamma(\Delta L_{t+1}/L_t)] = 0. \quad (11)$$

Now, assume that expectations are rational and denote the expectational error by $\epsilon_t$, with $E_t(\epsilon_{t+1}) = 0$. Then, after some rearrangement, the Euler equation becomes

$$[\beta_{t+1}p_{t+1}^l - p_t^l + \alpha_0[\beta_{t+1} - 1] + \alpha_1[\beta_{t+1}IK_{t+1} - IK_t + (\beta_{t+1}IK^{2}_{t+1})/2] + \gamma[\beta_{t+1}(\Delta L_{t+1}/L_t) - (\Delta L_t/L_{t-1})] + \theta[\beta_{t+1}Y_{t+1}/K_t] = \epsilon_{t+1}, \quad (12)$$

where $\beta_{t+1} = \beta_{t+1}(1 - \delta)$. Because we treat $\beta_{t+1}$ and $\delta$ as data, this equation is linear in the four structural parameters to be estimated: $\alpha_0, \alpha_1$, and $\gamma$ from the adjustment cost function, plus the parameter $\theta$ from the production function.

2.2. Estimation

We estimated the Euler equation using the Generalized Method of Moments (GMM) procedure described in Hansen and Singleton (1982). To set up notation that will be useful later, we briefly describe the GMM procedure. To begin, let $b = [\alpha_0, \alpha_1, \gamma, \theta]$, the coefficient vector. Then, the Euler equation in (12) can be written simply as

$$f(b) = \epsilon_{t+1}, \quad (13)$$
where we have suppressed the dependence of \( f(\cdot) \) on the variables. Further, denote the instrument set by \( Z_t \), which is a \( q \times 1 \) vector. Under rational expectations, these instruments should be orthogonal to the expectational error, \( \varepsilon_{t+1} \), so that \( E[Z_t f(b)] = 0 \). The sample counterpart of this set of \( q \) population moments is

\[
g_r(b) = (1/T) \sum_{t=1}^{T} Z_t f(b),
\]

where \( T \) is the number of observations in the full sample. (The subscript 'r' refers to 'restricted', a notation that will be useful below.) The GMM estimate is defined as the value of \( b \) that minimizes

\[
J_r(b) = [g_r(b)]' W_r [g_r(b)],
\]

where \( W_r \) is the optimal weighting matrix described in Newey and West (1987a). The \( J \)-statistic, used for testing the model's overidentifying restrictions, equals the number of observations \( (T) \) multiplied by the minimized value of (15). This statistic is asymptotically distributed as \( \chi^2 \) with degrees of freedom equal to the number of instruments minus the number of parameters.

Before proceeding to estimation, we need to define our instrument set, \( Z_t \). For the GMM estimates to be consistent, the instruments must be uncorrelated with the error term in the Euler equation. Given the assumption of rational expectations, any variable in the firm's information set at time \( t \) would be uncorrelated with the expectational error \( \varepsilon_{t+1} \). This reasoning implies that any variable dated \( t - 1 \) or earlier would be a valid instrument, provided that the error term in the Euler equation reflects expectational error and nothing more. However, to protect ourselves against certain forms of measurement error that could lead to inconsistent estimates, we restrict our instrument set to variables dated \( t - 2 \) and earlier. Accordingly, \( Z_t \) is specified to be a 13-element vector consisting of

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\(^5\) This weighting matrix yields a positive definite covariance matrix for the parameter estimates that is robust to heteroskedasticity and serial correlation of unknown type. Following a suggestion in Newey and West (1987a), the lag length for the covariance matrix was always set equal to the sample size raised to the one-third power.

\(^6\) To be specific, consider classical measurement error where \( x_t \) equals the true value, \( x_t^\ast \), plus white noise, \( \eta_t \). If \( x_t \) enters the Euler equation linearly, \( \eta_t \) is incorporated into the equation's error term. Because our Euler equation includes variables dated as early as \( t - 1 \), the error term could include \( \eta_{t-1} \), so valid instruments would have to be dated \( t - 2 \) or earlier. Of course, even with this early dating, consistency is not guaranteed if the measurement error is serially correlated or if it affects \( I K_t \), which enters the Euler equation as a quadratic.

Because investment Euler equations typically have been estimated with variables dated \( t - 1 \) as instruments, we also performed all of our empirical work using an instrument set that included the first and second lags of each variable in \( Z_t \). The results with this alternative instrument set were not appreciably different from those reported below.
Table 1
Estimates of the Euler equation for business equipment over 1960:1 to 1991:3 (standard errors in parentheses)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$\gamma$</th>
<th>$\theta$</th>
<th>$J$-statistic</th>
<th>Marginal sig. level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>-0.760</td>
<td>1.168</td>
<td>0.872</td>
<td>0.042</td>
<td>9.9</td>
<td>0.36</td>
</tr>
<tr>
<td>(0.187)</td>
<td>(1.93 )</td>
<td>(0.695)</td>
<td>(0.020)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a constant and the second and third lags of $p_t^r, IK_t, IK_t^2, Y_t/K_{t-1}, \Delta L_t/L_{t-1}$, and $\beta_t$.

We estimate Eq. (12) for the United States using data from 1960:1 to 1991:4 on the discount rate; on aggregate business sector output and employment; and on investment, capital stock, the tax-adjusted purchase price, and the depreciation rate for business equipment. The Appendix discusses the construction and data sources for each series. The Euler equation— which requires one lead of data for expected variables—is estimated from 1960:1 to 1991:3, with the results shown in Table 1. Of the four parameters, we have theoretical priors on $a_1$ and $\theta$. The point estimates of both parameters are positive, consistent with these priors. However, the standard error of $a_1$ is quite large, and the coefficient is not significantly different from zero. In addition, the parameter $\theta$—which should equal the income share of equipment—is somewhat smaller than might be expected. The $J$-statistic does not reject the overidentifying restrictions, suggesting that the orthogonality assumptions of the model are satisfied. However, as noted in the introduction, the $J$-statistic is a weak test of overall model adequacy; thus, we embark on a more complete examination of model specification.

3. Split-sample tests of structural stability

The preceding section presented GMM estimates of a standard investment Euler equation, accompanied by the $J$-statistic. This information constitutes the full empirical analysis—including both model estimation and validation—undertaken by all previous studies in this area. In contrast, we take these full-sample

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7 We excluded nonresidential structures from our empirical Euler equation because of the poor performance of most models for this type of capital (see, for example, Oliner, Rudebusch, and Sichel, 1995).
estimates as merely a starting point for a more thorough investigation of the adequacy of the Euler equation. In this section, we focus on split-sample tests of structural stability. For a sample of $T$ observations and a breakpoint at time $T_1$, the first subsample contains the $T_1$ observations dated $t = 1, \ldots, T_1$ and the second subsample contains the $T_2$ observations dated $t = T_1 + 1, \ldots, T$, with $T = T_1 + T_2$. We employ three split-sample tests to evaluate structural stability across these two partial samples. The first two are the Wald test and D test, which were described by Newey and West (1987b). The third test was proposed by Ghysels and Hall (1990b); we refer to this as the GH test. After describing the general form of these tests, we present the test results for the investment Euler equation.

3.1. Three tests

3.1.1 Wald test

To test for a structural shift, the Wald test compares the model estimates from each of the two subsamples. Let the vectors $b_1$ and $b_2$ denote the parameter values for the first and second subsamples, respectively. To obtain estimates of these parameters, we define the unrestricted model using a dummy variable, denoted $d_t$, that equals zero in every period through the breakpoint and equals unity in every period after the breakpoint. With this notation, the unrestricted Euler equation can be written as

$$(1 - d_t)f(b_1) + d_t f(b_2) = \varepsilon_{t+1},$$

or to simplify further,

$$f_u(b_1, b_2) = \varepsilon_{t+1}. \tag{17}$$

To estimate $b_1$ and $b_2$, we use the same 13 instruments, denoted $Z_{it}$, as for the full-sample estimation. Analogous to Eq. (14) above, the sample moment conditions for the unrestricted model are

$$g_u(b_1, b_2) = (1/T) \sum_{t=1}^{T} Z_t f_u(b_1, b_2). \tag{18}$$

The GMM estimates $(\hat{b}_1, \hat{b}_2)$ of the unrestricted model are defined as the values of $b_1$ and $b_2$ that minimize

$$J_u(b_1, b_2) = [g_u(b_1, b_2)]'W_u[g_u(b_1, b_2)], \tag{19}$$

where $W_u$ is the Newey–West weighting matrix for the unrestricted model.

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8 Also see the discussion in Andrews and Fair (1988).
9 Throughout our analysis, we do not vary our instrument set; this issue is considered further in Section 3.2 below.
With these unrestricted model estimates, the Wald statistic for testing the null hypothesis that $b_1 = b_2$ is a quadratic form in the difference between $\hat{b}_1$ and $\hat{b}_2$. Namely,

$$WALD = T \left[ \hat{b}_1 - \hat{b}_2 \right]'M[\hat{b}_1 - \hat{b}_2],$$

where $M$ is the appropriate weighting matrix for the quadratic form. See Newey and West (1987b) for details. Under the null hypothesis, this test has an asymptotic $\chi^2$ distribution, with degrees of freedom equal to the number of restrictions tested. For our application, $b_1 = b_2$ implies four restrictions.

### 3.1.2. D test

The form of the D test is fairly intuitive. Loosely speaking, the test statistic equals the difference between the $J$-statistics obtained from the restricted and unrestricted models. Thus, the D test has the same form as a likelihood ratio test, because the test statistic is the difference between the optimal values of a restricted and an unrestricted objective function. The unrestricted estimates, in which the parameters are allowed to differ across the two subsamples, are exactly $(\hat{b}_1, \hat{b}_2)$ described above for the Wald test. For the restricted model, in which parameters do not differ across subsamples, we need estimates of $b$ from Eq. (13).

It might seem appropriate to obtain the restricted estimates by minimizing the objective function in (15), which is repeated here:

$$J_r(b) = [g_r(b)]'W_r[g_r(b)].$$

However, as Newey and West (1987b) point out, the restricted and unrestricted GMM estimates for the D test must be obtained by using the same weighting matrix. For example, one alternative we explore used the unrestricted weighting matrix $W_u$ for both estimates. In this case, the restricted GMM parameter vector, $\hat{b}$, is the value of $b$ that minimizes a hybrid quadratic form

$$J_h(b) = [g_r(b)]'W_u[g_r(b)],$$

where $W_u$ is the same matrix used in Eq. (19) above. Let $J_d(\hat{b})$ denote the minimized value of the restricted GMM objective function in (22) and let $J_d(\hat{b}_1, \hat{b}_2)$ denote the minimized value of the unrestricted GMM objective function in (19). Then, the D test for this choice of weighting matrix is defined as the sample size multiplied by the difference between the restricted and unrestricted minimands:

$$D = T [J_h(\hat{b}) - J_d(\hat{b}_1, \hat{b}_2)].$$

Under the null hypothesis that $b_1 = b_2$, this test statistic has the same asymptotic distribution as the Wald statistic, a $\chi^2$ distribution with degrees of freedom equal to the number of parameters tested for constancy.
In constructing the D test, we also explored the obvious alternative of using the restricted weighting matrix $W$, in constructing both the unrestricted and restricted GMM estimates. Issues relating to the choice of the weighting matrix are discussed further below.

3.1.3. GH test

The tests above focus on the restricted and unrestricted parameter vectors. In contrast, Ghysels and Hall (1990b) have proposed a test for the structural stability of models estimated by GMM that focuses on the moment conditions across subsamples. Their test estimates the model over the first subsample and then examines whether the moment conditions of the model are satisfied over the second subsample using the parameter estimates obtained in the first subsample.

Specifically, the null hypothesis of structural stability for the Ghysels and Hall (GH) test is

$$H_0: \quad E[Z_t f(b_1)] = 0 \quad \text{for} \quad t \in [1, T_1],$$
$$E[Z_t f(b_1)] = 0 \quad \text{for} \quad t \in [T_1 + 1, T].$$

This null specifies that the population moment conditions delivered by the Euler equation and the instruments hold over both subsamples for the same set of coefficients. The alternative hypothesis is

$$H_1: \quad E[Z_t f(b_1)] = 0 \quad \text{for} \quad t \in [1, T_1],$$
$$E[Z_t f(b_1)] \neq 0 \quad \text{for} \quad t \in [T_1 + 1, T].$$

That is, under the alternative hypothesis, the set of parameters that satisfy the moment conditions of the model in the first subsample do not satisfy those conditions in the second subsample.

To develop a formal test, define the sample moment conditions for the first subsample as

$$g_1(b_1) = \frac{1}{T_1} \sum_{t=1}^{T_1} Z_t f(b_1).$$

The vector of GMM estimates generated by these conditions is $\hat{b}_1$ (which, of course, is the same as the estimates obtained from the unrestricted model above over the first subsample). Next, define $g_2(\hat{b}_1)$ as the sample moment conditions for the second subsample evaluated at the parameter estimates from the first

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10This test was independently proposed in Hoffman and Pagan (1989).
subsample. That is,
\[ g_2(\hat{\beta}_1) = \frac{1}{T_2} \sum_{t=T_1+1}^{T} Z_t f(\hat{\beta}_1). \]

Using this notation, the GH test statistic is
\[ GH = T_2 \left[ g_2(\hat{\beta}_1) \right] \hat{\Sigma}^{-1} \left[ g_2(\hat{\beta}_1) \right], \]
where \( \hat{\Sigma} \) is a consistent estimator of the weighting matrix \( V \), which is described further below.

The intuition behind the GH test is clear: if the null hypothesis is true, the parameter estimates from the first subsample should nearly satisfy the moment conditions over the second subsample—that is, \( g_2(\hat{\beta}_1) \) should be close to zero. If \( g_2(\hat{\beta}_1) \) is not close to zero, then some aspect of the structure of the model must have changed across the two subsamples.\(^{11}\) The GH test is asymptotically distributed \( \chi^2 \) with degrees of freedom equal to the number of elements of \( g_2 \), that is, the number of moment conditions that are examined over the second subsample. In our application with 13 instruments, the test has 13 degrees of freedom.

The matrix \( \Gamma \) is defined as
\[ \Gamma = W_2^{-1} + cD_2 \left[ D_1 W_1 D_1 \right]^{-1} D_2, \]
where \( W_1 \) and \( W_2 \) are estimates of the GMM weighting matrices for the first and second subsamples, \( c = T_1 / T_2 \), and \( D_1 \) and \( D_2 \) are the derivatives of \( g_1 \) and \( g_2 \) with respect to the parameter vector.

We consider three different estimates of \( V \), all of which have the same asymptotic distribution under the null hypothesis but take on different values in finite samples. Although all three versions of \( \hat{\Sigma} \) use estimates of \( D_1 \) and \( D_2 \) evaluated at \( \hat{\beta}_1 \), they employ different estimates of \( W_1 \) and \( W_2 \). The first estimate of \( V \), following Ghysels and Hall, obtains consistent estimates of both \( W_1 \) and \( W_2 \) using \( \hat{\beta}_1 \), so that the estimated weighting matrix for the second subsample relies on parameter estimates from the first subsample.\(^{12}\) We denote the resulting estimate of \( V \) by \( \hat{\Sigma}_1 \). The second estimate, denoted by \( \hat{\Sigma}_2 \), uses the \( \hat{W}_2 \) constructed for the Euler equation estimated over the second subsample. Thus, \( \hat{W}_1 \) is based on \( \hat{\beta}_1 \) and \( \hat{W}_2 \) is based on \( \hat{\beta}_2 \). The third alternative, \( \hat{\Sigma}_r \), replaces both \( \hat{W}_1 \) and \( \hat{W}_2 \) by \( \hat{W}_r \), the weighting matrix associated with the

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\(^{11}\) Note that the GH test is effectively an average of a set of estimated residuals. Dufour, Ghysels, and Hall (1994) propose an extension of the GH test that examines the information in individual residuals.

\(^{12}\) Ghysels and Hall refer to this test as the TS test. They argue that constructing this estimate of \( V \) provides some computational convenience because the model needs to be estimated only over the first subsample. In practice, we found this convenience to be negligible.
restricted model described above that estimates $\hat{\beta}$ over the full sample.\footnote{This final version of the GH test is very close to what Ghysels and Hall propose as the TSS test, which uses a hybrid estimate of $W$, based on $\hat{b}$,} Empirical results for all three versions of the GH test are presented below. As we shall show, the exact specification of the weighting matrix $\hat{V}$ has a large influence on the results obtained in our application.

3.2. Empirical results

Before examining the results for specific tests, two important details must be addressed: the choice of a breakpoint and the choice of an instrument set. For the Wald and D test, we move the breakpoint sequentially through the sample and compute a test statistic for each breakpoint. The breakpoint with the highest value of the test statistic can be used to test whether the model's parameters are constant across the subsamples defined by this breakpoint. However, because the data are used to determine this breakpoint, the maximal test statistic will not have a conventional distribution. Instead, it is necessary to use the distribution worked out by Andrews (1993) for maximal F-type tests. Unfortunately, for the GH test, the distribution of such a maximal test statistic is unknown; thus, we select a priori a mid-sample breakpoint and calculate the GH statistic—a case for which the usual distribution theory is applicable—and then compute a sequence of GH test statistics over the sample to check whether the mid-sample value is representative.

Although general efficiency arguments, such as those in Hansen, Heaton, and Ogaki (1988), can be made regarding the choice of an instrument set, little practical guidance is available for finite samples. For our entire analysis, we maintained the same instrument set used in the full-sample estimation. As an alternative, our implementation of the Wald test and the D test could have used an instrument set including the dummy variable $d_i$ or the dummy variable interacting with other instruments. Furthermore, for the GH test we could have used completely different variables as instruments for each of the subsamples.\footnote{As noted by Ghysels and Hall (1990a), different instruments in the second sample may be chosen in order to construct a test that is more powerful against departures from the null in particular directions.}

3.2.1. Wald test

The top panel of Fig. 1 plots the value of the Wald statistic for each period using that period as the breakpoint. We compute test statistics for every possible breakpoint from 1964:2 to 1987:2, which corresponds to the middle 70 percent
Fig. 1. Wald and D test statistics for equipment Euler equation (four degrees of freedom).
of the sample. If the breakpoint were specified a priori, the conventional critical value for a \( \chi^2 \) statistic with four degrees of freedom would be appropriate; at the 5 percent significance level, this critical value is 9.5. However, for the sequence of Wald test statistics over all breakpoints, the distribution worked out by Andrews (1993) is more appropriate. For our range of breakpoints, the asymptotic critical values for the maximal test statistic are 16.5 at the 5 percent level and 20.7 at the 1 percent level. As shown, the Wald statistic exceeds 20 for a range of breakpoints around the early 1970s. Thus, even using Andrew's critical values, the Wald test provides strong evidence of parameter nonconstancy in the Euler equation.

3.2.2. \( D \) test

Results for the sequential \( D \) test are shown in the bottom panel of Fig. 1. Because the \( D \) test has the same asymptotic distribution as the Wald test under the null, the critical values noted above are applicable. As mentioned above, the restricted and unrestricted GMM estimates for the \( D \) test must be obtained using the same weighting matrix; we examined two alternatives: the weighting matrix from the unrestricted model (\( \hat{W}_u \)) and the weighting matrix from the restricted model (\( \hat{W}_r \)). Of course, this choice does not affect the asymptotic distribution of the test statistic because, under the null, both \( \hat{W}_u \) and \( \hat{W}_r \) are consistent estimates of the weighting matrix. However, as a numerical matter, the choice of weighting matrix turned out to make a sizable difference in our application. In particular, as shown in the chart, using the restricted matrix—rather than the unrestricted matrix—yielded lower values for the \( D \) test. The \( D \) test with \( \hat{W}_u \) rejects structural stability for most breakpoints near the middle of the sample, while the \( D \) test with \( \hat{W}_r \) never rejects.

We suspect that the numerical differences for the \( D \) test reflect differences in power of the two versions of the test. In particular, under the alternative hypothesis \((b_1 \neq b_2)\), the unrestricted weighting matrix (\( \hat{W}_u \)) remains a consistent estimate of the weighting matrix, but \( \hat{W}_r \) would be inconsistent. Therefore, a \( D \) test based on \( \hat{W}_u \) may well have better power against alternatives of parameter nonconstancy than tests based on the restricted matrix, \( \hat{W}_r \).

15 The choice of this range is somewhat arbitrary but is recommended by Andrews (1993) on the basis of a Monte Carlo experiment that explores the trade-off between the length of the range and test power.

16 For our model, which is linear in the parameter vector, the values of the Wald statistic and the \( D \)-statistic based on the unrestricted weighting matrix are the same. See Newey and West (1987b) for further discussion of this point.

17 On the other hand, as suggested by a referee, the unrestricted parameter estimates may overfit the data in small samples and make the standard errors too tight. In this case, the unrestricted weighting matrix tests may overreject under the null (i.e., a size distortion).
3.2.3. GH test

For the GH test, we first split the sample at its midpoint, placing all observations through the second quarter of 1975 in the first subsample. With a midpoint break, the value of the GH statistic using \( \hat{V}_1 \) is 8.6, well below the critical value of 22.4 for a \( \chi^2 (13) \) test at the 5 percent significance level. However, the GH test statistic using \( \hat{V}_2 \) – which has the same asymptotic distribution as the statistic using \( \hat{V}_1 \) – equaled 22.5, just above the critical value. Finally, the GH test using \( \hat{V}_r \) provides a test statistic of 7.4, less than the critical value. The numerical differences in these test statistics indicate that the GH test can be very sensitive to the choice of a weighting matrix, as is the D test.

As discussed by Ghysels and Hall (1990b), the GH test can be specialized to focus on a single moment condition. For the mid-sample split, we computed such specialized GH tests for each of the 13 moment conditions (using each of the three estimates of \( V \)). These tests suggested that no single moment condition was particularly responsible for the rejection in the joint test of all moments.

As indicated above, the lag length used for the covariance matrix was always set equal to the sample size raised to the one-third power. We also tried the lag lengths chosen by the optimal lag selection procedure in Newey and West (1992). These, however, varied substantially as the break-point moved through the sample, raising the possibility that there may not be enough observations for this procedure to have optimal properties. With such lags, the D and Wald tests no longer rejected parameter constancy, but the results from the GH test were essentially unchanged.
On balance, the GH test with a mid-sample split provides some evidence of structural instability. In order to check the robustness of the test results, we also computed the three versions of the GH test for all breakpoints in the middle 70 percent of the sample. The resulting sequence of statistics for the three versions of the GH test are shown in Fig. 2. Strictly speaking, the distribution of these sequences is unknown, as the Andrews results do not generalize to the GH test. However, the sequences shown can indicate whether the mid-sample results are atypical. The results for the GH statistic based on \( \hat{V}_2 \) appear to confirm the structural instability suggested by the mid-sample test.

4. Subsample parameter estimates

While the preceding section provided evidence of structural instability in the Euler equation, the results were sensitive to the choice of the weighting matrix. In this section, we provide more evidence on the parameter nonconstancy of the Euler equation using a complementary but less formal technique. We diagnose parameter nonconstancy by examining how subsample parameter estimates change as additional observations are included in the estimation period. The resulting sequence of subsample model estimates provides clear, intuitive information about possible instabilities.\(^{20}\)

To implement this technique, we first estimate the Euler equation over the 15-observation subsample from 1960:1 to 1963:3, then add one observation, reestimate, add another observation, reestimate, and so on. To gauge whether the movements in the parameter estimates are large enough to indicate nonconstancy, we include a confidence bound for each coefficient equal to plus and minus two standard errors (as estimated over the subsample for that coefficient). If the parameter estimates are stable, then each estimate should remain inside the band plotted up to that point. Further, if a parameter exhibits constancy, additional observations should improve the efficiency of the parameter estimates, implying that the confidence bands should narrow with the addition of new observations.

The first two panels of Fig. 3 display the sequence of estimates for the adjustment cost parameters, \( \alpha_0 \) and \( \alpha_1 \). As the top panel shows, the linear adjustment cost parameter, \( \alpha_0 \), initially drifts down and then reverses course once the subexamples extend into the late 1960s. The estimates at

\(^{20}\) The classic reference for sequential estimation of a linear model over partial samples is Brown, Durbin, and Evans (1975) (also see Dufour, 1982). Such sequences of parameter estimates are often referred to as recursive estimates, a term we eschew because we do not employ a recursive algorithm for estimation.
Fig. 3. Subsample estimates for equipment Euler equation (dashed lines show bands of plus and minus two standard errors for each coefficient).
Labor interactive adjustment cost parameter (γ)

Production function parameter (θ)

Fig. 3 (continued)
the end of the sample are clearly outside the error bands from the earlier part of the sample, suggesting that this parameter is not constant over the sample. Similarly, the estimates of the quadratic adjustment cost parameter, $\alpha_1$, increase from about zero in the early part of the sample period to values above 10 for subsamples ending in the late 1960s and the early 1970s. This parameter then drifts back down toward zero with further lengthening of the estimation period. At several points, the values of $\alpha_1$ breach the error bands earlier in the sample period. In addition, the error bands do not narrow much as the sample is lengthened. Thus, we conclude that $\alpha_1$, like $\alpha_0$, does not exhibit constancy over the sample.

The next panel of Fig. 3 shows the sequence of estimates of the interactive parameter for labor and capital adjustment costs, $\gamma$. This parameter is volatile in the first part of the sample and then begins a gradual upward drift. Clearly, some of the values of the parameter breach earlier error bounds. In addition, the error bands do not contract later in the sample. The final panel of the chart shows estimates of the production function parameter, $\theta$. Exhibiting a pattern similar to $\alpha_0$, this parameter begins to drift upward in the late 1960s, eventually breaching its earlier bounds. Overall, the subsample evidence for both of these parameters suggests the presence of nonconstancy.

A word of caution is in order about the interpretation of the subsample estimates. The results in Fig. 3 could have been obtained even under the null of no structural change if there were small-sample biases in the estimates of the parameters and their standard errors. For example, it is possible that the asymptotic confidence bands simply understate the true variability of the parameter estimates early in the sample, leading one to conclude incorrectly that there had been structural change. Although a Monte Carlo simulation would be required to examine this possibility explicitly, it is worth noting that the rejections from the Wald and D test occur near the middle of the sample, at a point where small-sample problems are minimized.

5. Conclusion

Over a decade ago, Lucas and Sargent (1981, pp. 302–303) stated their view of the 'failure' of macroeconometrics:

[The] question of whether a particular model is structural is an empirical, not theoretical, one. If the macroeconometric models had compiled a record of parameter stability, particularly in the face of breaks in the stochastic behavior of the exogenous variables and disturbances, one would be skeptical as to the importance of prior theoretical objections of the sort we have raised. In fact, however, the track record of the major econometric models is ... very poor.
Our results suggest that this indictment may be apt for Euler equations as well. It appears that the Euler equation approach in investment has not, thus far, been a success when judged by the standards of the Lucas critique—arguably, the most relevant standards. In particular, an Euler equation representative of those in the applied investment literature exhibits a considerable degree of parameter instability. Indeed, in light of the results of Chirinko (1988) and Dufour (1989) that show only a modest amount of parameter nonconstancy in traditional reduced-form investment models such as the neoclassical model, the empirical Euler equation appears to provide no improvement when judged by the metric of structural stability.

We draw two major conclusions from our results. First, as a methodological note, our results recommend a major re-orientation in the evaluation of GMM estimates of Euler equations. Given the obvious inadequacy of the J-test as a test for model misspecification, parameter stability tests should routinely be reported for empirical Euler equations. Furthermore, the sensitivity of some of our results to small differences of technique that should make no difference asymptotically—for example, in the specification of weighting matrices—suggests that the asymptotic distribution theory that underpins the application of GMM may be misleading in small samples.

Second, as a substantive conclusion, we question whether Euler equations will be successful in uncovering structural parameters, at least for aggregate investment. Our evidence could be criticized on the grounds that our particular Euler equation is just an incorrectly specified model. However, the hard question, given the similarity of our model to earlier work, is whether any tractable Euler equation for investment can be found that is not misspecified. Surprisingly, there has been little skepticism about the modelling potential of the Euler equation approach to macroeconometrics. The tight restrictions imposed on dynamic structure by an Euler equation as well as the difficulties in rigorously aggregating across heterogeneous firms and heterogeneous types of capital, appear to make structural analysis a perilous exercise. We doubt that this challenge can be easily met.

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21 When we examined the parameter stability of a traditional neoclassical model in our data set, we found no more instability than for the Euler equation. However, in other work (Oliner, Rudebusch, and Sichel, 1995), we have found that in terms of out-of-sample predictive power, a metric quite sensitive to problems of structural adequacy, the traditional models perform much better than the Euler equation.

22 The few examples that we are aware of in this regard are West (1988), Ghysels and Hall (1990a), Epstein and Zin (1991), Osterberg (1992), Demers, Demers, and Schaller (1993), and Wirjanto (1993).

23 Indeed, see West and Wilcox (1994) on the inadequacy of the asymptotic distribution theory for finite samples.

24 Exceptions include Garber and King (1983), Blinder (1986), and Ando (1989).
Data appendix

Investment ($I$)
Quarterly spending on producers' durable equipment in billions of 1987 dollars, seasonally adjusted; from the U.S. National Income and Product Accounts (NIPAs).

Capital stock ($K$)
We interpolated a quarterly series from the annual net stock of private nonresidential equipment published by the Bureau of Economic Analysis (BEA). To do so, we set the fourth-quarter value of the interpolated series equal to the value of BEA's (year-end) net stock for that year. Then, to interpolate between year-end values, we assumed that the quarterly changes in the stock of equipment were proportional to the quarterly pattern of the corresponding investment outlays. The resulting quarterly capital stocks are measured in billions of 1987 dollars.

Employment ($L$)
Monthly employment on private nonfarm payrolls, in millions of persons, seasonally adjusted; from the Bureau of Labor Statistics. We averaged the monthly data to obtain a quarterly series.

Output ($Y$)
Quarterly gross domestic product in the nonfarm business sector excluding housing, in billions of 1987 dollars, seasonally adjusted; from the NIPAs.

Purchase price of capital goods ($p'$)
We constructed the following index of the after-tax purchase price of equipment:

\[ p'_t = \frac{PPDE_t}{p_t} \left[ \frac{1 - ITCE_t - \tau_t ZPDE_t (1 - W_t ITCE_t)}{1 - \tau_t} \right]. \]

(A.1)

$PPDE_t$ and $p_t$ are, respectively, the NIPA implicit price deflators for producers' durable equipment and gross domestic product in the nonfarm business sector excluding housing; both series are seasonally adjusted by the BEA. The remaining pieces of (A.1) are tax parameters: $\tau_t$ is the maximum corporate tax rate in effect during quarter $t$; $ITCE_t$ is the average rate of investment tax credit for equipment; $W_t$ is the percentage of the cost of equipment that cannot be depreciated if the firm takes the investment tax credit; and $ZPDE_t$ represents the present value of $\$1$ of depreciation allowances for equipment. $W_t$, $ITCE_t$, and $\tau_t$ were taken from the Federal Reserve Board's Quarterly Econometric Model;
see Brayton and Mauskopf (1985) for the construction of variables in the Quarterly Model. To compute $Z_{PDE}$, we discounted the stream of future tax depreciation allowances using the discount rate $(1 - \tau_r)R_B$, where $R_B$ is the interest rate on ten-year U.S. Treasury bonds. We used the Treasury rate, rather than an interest rate on corporate debt, because the tax allowances are (essentially) a risk-free income flow.

**Discount rate ($r$)**

$r_t$ measures the real opportunity cost of funds for the firm. Following the procedure in the Federal Reserve Board's Quarterly Model, we specified $r_t$ to be a weighted average of the returns to debt and equity:

$$r_t = \omega r_d + (1 - \omega) r_e,$$

with

$$r_d = (1 - \tau_t) R_T B_t - E P_B$$ and $r_e = 2 \cdot D I V P_t$.

We set the weight on debt, $\omega$, to 0.3; this relatively small debt share reflects the heavy reliance firms place on retained earnings as a source of funds. $R_T B_t$ is the yield on three-month Treasury bills; we use a short-term interest rate because the Euler equation (12) discounts terms dated at quarter $t + 1$ only back to quarter $t$. $\tau_t$ is (as above) the maximum corporate tax rate, and $E P_B$ is the expected inflation rate for the business output deflator $p_t$, computed as a geometrically-declining weighted average of the most recent 12 quarters of inflation in that series. $r_e$ equals two times the Standard and Poor's dividend–price ratio for common stocks; $D I V P$ is multiplied by two on the assumption that dividend payouts represent about one-half of the return to equity. The data for $D I V P$ and $\tau$ were taken directly from the Federal Reserve Board's Quarterly Model.

The resulting estimate of $r_t$ is at an annual rate. To compute the discount factor $\beta$ at a quarterly rate, we set $\beta = \left[1/(1 + r_t)\right]^{1/4}$.

**Depreciation rate ($\delta$)**

$\delta = 0.03784$. This is the quarterly depreciation rate for equipment specified in Bernanke, Bohn, and Reiss (1988).

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