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New and Old Models of Business Investment: A Comparison of Forecasting Performance

RECENT EMPIRICAL RESEARCH on investment has focused on the estimation of the stochastic first-order conditions, or Euler equations, from dynamic models derived under rational expectations. Because these models have an explicit structural interpretation, they are theoretically more appealing than traditional models of investment. However, the empirical performance of Euler-equation models has not been tested against the traditional models. This paper performs such a test by adding two Euler equations to the usual group of traditional investment models examined in previous studies—namely, the accelerator, neoclassical, modified neoclassical, and Q-theory models.[1]

The first of our two Euler equations is a "canonical" model that typifies the equations estimated in recent years.[2] Despite its popularity, this canonical model has a restrictive dynamic structure that is unlikely to capture the time lags inherent in the investment process. In contrast, our second Euler equation explicitly accounts for

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1. Earlier evaluations of investment models include Bernanke, Bohn, and Reiss (1988), Kopcke (1985, 1993), Clark (1979), Elliott (1973), Bischoff (1971), Jorgenson, Hunter, and Nadiri (1970a, b), and Jorgenson and Siebert (1968).

2. To our knowledge, Abel (1980) was the first to estimate an investment Euler equation under rational expectations. For later work within this literature, see Pindyck and Rotemberg (1983), Shapiro (1986a, b), Gilchrist (1990), Gertler, Hubbard, and Kashyap (1991), Hubbard and Kashyap (1992), Whited (1992), Carpenter (1992), and Ng and Schaller (1993).

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the lag between the start of an investment project and the later date at which the new capital begins to contribute to the firm's production. By embedding such "time-to-build" lags, which were emphasized by Kydland and Prescott (1982), this equation has a richer structure than most previous investment Euler equations.[3]

This paper focuses on the ability of the various models to forecast investment in equipment and in nonresidential structures. From a practical standpoint, such out-of-sample tests are needed to determine which models have the most value as forecasting tools. Moreover, beyond this practical objective, out-of-sample performance is a powerful test of model specification (see, for example, Hendry 1979). We conduct two sets of tests. The first set of tests examines the size, bias, and serial correlation of the models' one-step-ahead forecast errors, similar to the out-of-sample tests performed by Kopcke (1985) and Clark (1979). In addition, we compare the information content of model forecasts by regressing actual investment on predictions from pairs of models. Fair and Shiller (1990) have argued that such regressions provide a powerful test of alternative models.

To summarize the results, we find that the Euler equations produce extremely poor forecasts of investment for both equipment and nonresidential structures. The time-to-build version of the Euler equation outperforms the basic Euler equation in our tests, but the improvement is modest. All the Euler equations have mean squared forecast errors many times larger than those of the traditional models. Moreover, the Fair-Shiller tests suggest that, as a group, the traditional models for equipment dominate the Euler equations; for nonresidential structures, the Fair-Shiller tests show that neither the Euler equations nor the traditional models have any forecasting ability.

The paper is organized as follows. The next section describes the models in our horse race, while section 2 briefly discusses our data set. Section 3 presents full-sample estimates of each model, in order to gain some initial information on their relative fit. Section 4 documents that the Euler equations produce relatively poor forecasts, and section 5 attempts to explain why this is so, arguing that the standard assumptions that underlie these equations could be to blame. Section 6 concludes the paper and suggests areas for future research.

1. THE INVESTMENT MODELS

A. Two Investment Euler Equations

To derive our Euler equations of investment spending, we adopt several assumptions that are fairly standard in the literature:

- The firm's production function is Cobb-Douglas with constant returns to scale:

$$Y_t = F(K_{t-1}, L_t) = AK_{t-1}^{\theta} L_t^{(1-\theta)} \,, \tag{1}$$

3. Relatively few researchers have estimated structural time-to-build models of investment. This work appears to be limited to Chirinko and Schiantarelli (1991), Altug (1989), Rossi (1988), and Park (1984). For recent theoretical work on time-to-build models, see Altug (1993).

where Y_t and L_t are output and employment during period t, and K_{t-1} is the capital stock at the end of period $t-1$. The marginal product of capital is

$$F_{K_t} \equiv \partial Y_{t+1}/\partial K_t = \theta Y_{t+1}/K_t \, . \tag{2}$$

• Capital is a quasi-fixed factor subject to the usual quadratic adjustment costs, while employment is assumed to be a variable factor. Let I_t denote gross investment during period t. Then, the adjustment cost function is

$$C(I_t, K_{t-1}) = [\alpha_0(I_t/K_{t-1}) + (\alpha_1/2)(I_t/K_{t-1})^2]K_{t-1} \, .^4 \tag{3}$$

The partial derivatives of $C(I_t, K_{t-1})$ are

$$C_{I_t} = \alpha_0 + \alpha_1 IK_t \quad \text{and} \quad C_{K_{t-1}} = -(\alpha_1/2)IK_t^2 \, , \tag{4}$$

where $IK_t \equiv I_t/K_{t-1}$. For the firm's investment decision to be well defined, C_I must be increasing with the level of investment; that is, $\partial C_{I_t}/\partial I_t = \alpha_1/K_{t-1}$ must be greater than zero, implying that $\alpha_1 > 0$. We have no prior on the sign of α_0.

• All markets are perfectly competitive, implying that the price of output, the price of capital goods, and the wage rate are exogenous. We normalize both input prices by the price of output (p_t) and denote the resulting real price of capital goods and real wage by p_t^I and w_t, respectively.[5]

• The firm's discount rate is exogenous, so that financing decisions are irrelevant for the optimal investment path. We denote the firm's time-varying discount rate by r_t and the corresponding discount factor by $\beta_t = 1/(1 + r_t)$.

• There is only one type of capital, with a constant depreciation rate of δ. As discussed below, we relax this assumption in our empirical work by estimating separate equations for equipment and nonresidential structures.[6]

• Investment projects are subject to time-to-build lags, where our specification of these lags follows Taylor (1982). Let S_t denote the value of projects started in period t. All projects take τ periods to complete, so that additions to the capital stock in period t equal project starts in period $t - \tau$. The equation of motion for the capital stock is then

$$K_t = (1 - \delta)K_{t-1} + S_{t-\tau} \, . \tag{5}$$

4. If, instead, we were to specify a more general adjustment cost function that included interactions between fixed capital and employment, the investment Euler equations would include terms for the change in employment. However, these terms would introduce information into the Euler equations that is absent from the traditional models, undercutting our aim to compare limited-information models of investment.

5. The assumption of perfect competition in output markets could be relaxed—and indeed has been in other work on investment Euler equations. However, because the neoclassical and Q models that we estimate assume perfect competition, we make this assumption when deriving the Euler equation to enforce a degree of consistency across models.

6. To simplify the notation, we also ignore the role of taxes in this section. However, corporate tax provisions are incorporated in our empirical measure of the price of capital goods.

Further, let ϕ_i denote the proportion of the project's total value that is put in place i periods after the start, with $\phi_0, \ldots, \phi_\tau \geq 0$ and $\Sigma_{i=0}^{\tau} \phi_i = 1$. Thus, I_t equals the value put in place during period t from all projects underway at that time:

$$I_t = \sum_{i=0}^{\tau} \phi_i S_{t-i} . \tag{6}$$

Given this setup, we assume that the firm maximizes the expected present value of real future profits,

$$V_t = E_t \left(\sum_{s=t}^{\infty} \beta^*_{t,s} \, \pi_s \right) \tag{7}$$

where $\beta^*_{t,s} = \Pi_{j=t+1}^{s} \beta_j$ is the discount factor from period s back to period t and

$$\pi_s = F(K_{s-1}, L_s) - C\left(\sum_{i=0}^{\tau} \phi_i S_{s-i}, K_{s-1} \right) - w_s L_s - p_s^I \left(\sum_{i=0}^{\tau} \phi_i S_{s-i} \right) \tag{8}$$

(using equation (6) to represent I_s). Real profits equal revenue minus adjustment costs, labor costs, and the cost of purchasing new capital. Firms maximize (7) by choosing S_s, K_s, and L_s for all $s \geq t$, subject to equation (5), the capital-stock constraint. To carry out this maximization, we define the Lagrangian

$$\mathcal{L}_t = E_t \left[\sum_{s=t}^{\infty} \beta^*_{t,s} (\pi_s - \lambda_s (K_s - (1 - \delta) K_{s-1} - S_{s-\tau})) \right] . \tag{9}$$

Setting $\partial \mathcal{L}_t / \partial x_s =$ for all s, with $x_s = (S_s, K_s, L_s)$, yields a set of first-order conditions for each value of s. The two conditions needed to derive the Euler equation are

$$S_t\colon \quad \sum_{i=0}^{\tau} \phi_i E_t (\beta^*_{t,t+i} (p^I_{t+i} + C_{I_{t+i}})) = E_t (\beta^*_{t,t+\tau} \lambda_{t+\tau}) \tag{10}$$

$$K_{t+\tau}\colon E_t (\beta^*_{t,t+\tau+1} (F_{K_{t+\tau}} - C_{K_{t+\tau}})) = E_t (\beta^*_{t,t+\tau} \lambda_{t+\tau} - (1 - \delta) \beta^*_{t,t+\tau+1} \lambda_{t+\tau+1}) . \tag{11}$$

At the optimal level of starts, equation (10) says that the expected cost of acquiring and installing capital goods over the next τ periods ($p^I_{t+i} + C_{I_{t+i}}$, for $i = 0, \ldots, \tau$) equals the expected shadow value of the marginal addition to the capital stock when the project comes on line ($\lambda_{t+\tau}$). Both the cost of the project and its shadow value are discounted back to period t in this comparison. Equation (11) relates the shadow value of capital to its expected marginal return net of adjustment costs ($F_K - C_K$).

To derive the Euler equation, combine equations (10) and (11) to eliminate the terms in λ:

$$E_t(\beta^*_{t,t+\tau+1}(F_{K_{t+\tau}} - C_{K_{t+\tau}})) - \sum_{i=0}^{\tau} \phi_i E_t(\beta^*_{t,t+i}(p^I_{t+i} + C_{I_{t+i}}))$$

$$+ \sum_{i=0}^{\tau} (1 - \delta)\phi_i E_t(\beta^*_{t,t+i+1}(p^I_{t+i+1} + C_{I_{t+i+1}})) = 0 \,. \tag{12}$$

Now, assume that expectations are rational and let ϵ_{t+i} represent the expectational error for the terms in (12) dated at period $t + i$, with $E_t(\epsilon_{t+i}) = 0$. In addition, substitute the expressions for F_K, C_I, and C_K from equations (2) and (4) into (12). Then, after some rearrangement, we obtain

$$\sum_{i=0}^{\tau} \phi_i(\tilde{\Delta} p^I_{t+i+1}) + \sum_{i=0}^{\tau} \alpha_0 \phi_i(\tilde{\Delta}\beta^*_{t,t+i+1}) + \alpha_1(\beta^*_{t,t+\tau+1} IK^2_{t+\tau+1}/2)$$

$$+ \sum_{i=0}^{\tau} \alpha_1 \phi_i(\tilde{\Delta} IK_{t+i+1}) + \theta\left(\beta^*_{t,t+\tau+1} \frac{Y_{t+\tau+1}}{K_{t+\tau}}\right) = \sum_{i=1}^{\tau+1} \epsilon_{t+i} \tag{13}$$

where

$$\tilde{\Delta} p^I_{t+i+1} \equiv (1 - \delta)\beta^*_{t,t+i+1} p^I_{t+i+1} - \beta^*_{t,t+i} p^I_{t+i}$$

$$\tilde{\Delta}\beta^*_{t,t+i+1} \equiv (1 - \delta)\beta^*_{t,t+i+1} - \beta^*_{t,t+i}$$

$$\tilde{\Delta} IK_{t+i+1} \equiv (1 - \delta)\beta^*_{t,t+i+1} IK_{t+i+1} - \beta^*_{t,t+i} IK_{t+i} \,.$$

Most variables enter the Euler equation in the quasi-differenced form indicated by the $\tilde{\Delta}$ symbol. Because we treat δ and the β^* terms as data, all expressions in parentheses in (13) can be computed prior to estimation. This leaves $\tau + 3$ structural parameters to be estimated: α_0 and α_1 from the adjustment cost function, θ from the production function, and $\phi_0, \ldots, \phi_{\tau-1}$ to account for time-to-build lags (we restrict ϕ_τ to equal $1 - \Sigma_{i=0}^{\tau-1} \phi_i$).

The above Euler equation with time-to-build lags has a richer dynamic structure than is usually found in formal models of investment. The more typical specification arises as a special case of equation (13) when the time-to-build lag is zero: $\tau = 0$, $\phi_0 = 1$, and $\phi_i = 0$ for $i > 0$. In this case, (13) reduces to a simpler and more familiar equation:

$$(\tilde{\Delta} p^I_{t+1}) + \alpha_0((1 - \delta)\beta_{t+1} - 1) + \alpha_1\left(\frac{\beta_{t+1}}{2} IK^2_{t+1} + \tilde{\Delta} IK_{t+1}\right)$$

$$+ \theta\left(\beta_{t+1} \frac{Y_{t+1}}{K_t}\right) = \epsilon_{t+1} \tag{14}$$

where we have used the facts that $\beta^*_{t,t+1} = \beta_{t+1}$ and $\beta^*_{t,t} = 1$. As shown, (14) is a linear equation in three structural parameters: α_0, α_1, and θ. We examine the forecast performance of both the Euler equation with time-to-build lags [equation (13)] and the simpler version that omits time-to-build [equation (14)].

B. Four Traditional Models of Investment

Several well-known models of investment predate the Euler-equation approach: The Q model, the accelerator model, Jorgenson's neoclassical model, and the modified neoclassical model. Each of these models was analyzed in the comparative studies done by Clark (1979) and Bernanke, Bohn, and Reiss (1988). Our specification of each model is the same as in Clark (1979), except that we scale investment by the capital stock rather than by potential output.

Our traditional Q model takes the form:

$$IK_t = \psi + \sum_{s=0}^{N} \omega_s Q^A_{t-s}, \tag{15}$$

where Q^A is average Q, the ratio of the firm's market value to the replacement cost of its capital stock. Equation (15), although often estimated in the literature, is not a structural model. If we ignore time-to-build lags, the structural Q equation implied by our framework is

$$IK_t = -\alpha_0/\alpha_1 + (1/\alpha_1)(\lambda_t - p^I_t) = -\alpha_0/\alpha_1 + (1/\alpha_1)Q^A_t \ , \tag{16}$$

where the second equality replaces marginal $Q(\lambda_t - p^I_t)$ with average Q.[7] As can be seen from (16), the structural Q equation from a standard dynamic framework admits no role for lags of Q^A. These lags have been included by empirical researchers simply to improve the fit of the equation. The structural Q equation that emerges when we take into account time-to-build bears even less resemblance to equation (15).[8] Thus, we regard (15) as a reduced-form equation relating investment to prices in financial markets.

For both the accelerator model and Jorgenson's neoclassical model, the investment equation has the form:

$$I_t = \psi + \sum_{s=0}^{N} \tilde{\omega}_s \Delta K^*_{t-s} + \delta K_{t-1}, \tag{17}$$

where K^*_t is the firm's desired capital stock. In the accelerator model, the desired capital stock K^*_t is assumed proportional to output Y_t, so that $\Delta K^*_t = \zeta \Delta Y_t$. If we make this substitution for ΔK^* in equation (17), scale both sides of (17) by K_{t-1}, and add the random error u_t, we obtain:

7. To derive the first equality in (16), set $\tau = 0$ and $\phi_0 = 1$ in equation (10) and use (4) to substitute for C_t. The second equality follows from Hayashi (1982), who showed that average Q equals marginal Q under constant returns to scale and competitive markets.

8. See Oliner, Rudebusch, and Sichel (1993) for details.

$$IK_t = \delta + \frac{\psi}{K_{t-1}} + \sum_{s=0}^{N} \omega_s \frac{\Delta Y_{t-s}}{K_{t-1}} + u_t \, . \tag{18}$$

In contrast, Jorgenson's neoclassical model sets the marginal product of capital in a Cobb-Douglas technology equal to its real one-period rental price (c), so that $K_t^* = \theta(Y/c)_t$, and

$$IK_t = \delta + \frac{\psi}{K_{t-1}} + \sum_{s=0}^{N} \omega_s \frac{\Delta(Y/c)_{t-s}}{K_{t-1}} + u_t \, . \tag{19}$$

Finally, the modified neoclassical model, originated by Bischoff (1971), relaxes the symmetric treatment of output and the rental price in the neoclassical model. Bischoff assumed that capital is "putty-clay": Firms can choose the factor proportions for each new vintage of capital, but this choice is irreversible once the vintage has been installed. As discussed more fully in Oliner, Rudebusch, and Sichel (1993), the putty-clay assumption implies the following investment equation:

$$IK_t = \frac{\psi}{K_{t-1}} + \sum_{s=0}^{N} \left[\omega_{1s} \frac{(1/c)_{t-s-1} Y_{t-s}}{K_{t-1}} + \omega_{2s} \frac{(Y/c)_{t-s-1}}{K_{t-1}} \right] + u_t \, . \tag{20}$$

The ω_{2s} coefficients in equation (20) are expected to be negative, while ω_{1s} and the distributed lag coefficients for the other traditional models are expected to be positive.

2. DATA

We estimate equations (13), (14), (15), (18), (19), and (20) with quarterly data for the aggregate private business sector in the United States. These data cover the period 1952:1 to 1992:4. To estimate the traditional models, we require series for gross investment (I), capital stock (K), output (Y), the real user cost of capital (c), and average Q (Q^A). To estimate the Euler equations, we employ the same constant-dollar series for I, K, and Y, along with series for the real after-tax price of investment goods (p^I) and the discount factor (β), and an assumed value for the depreciation rate (δ). Here, we briefly describe the data, while the appendix in Oliner, Rudebusch, and Sichel (1993) fully documents each series.

Our constant-dollar series for output and gross investment are from the National Income and Product Accounts; because we estimate separate equations for producers' durable equipment and nonresidential structures, the investment data are disaggregated into these two categories. For each investment series, the corresponding capital stock is the annual constant-dollar net stock from the Bureau of Economic Analysis, which we interpolate to a quarterly frequency. To construct the real user cost of capital (c) and the real after-tax purchase price of investment goods (p^I), we follow the methodology in the Federal Reserve Board's Quarterly Econometric

Model. Our measure of average Q is based on the tax-adjusted formulation in Bernanke, Bohn, and Reiss (1988). The quarterly discount factor for the Euler equations is $\beta = 1/(1 + r)$, where r is an unweighted average of the real interest rate on three-month Treasury bills and the real return on equity; we use a short-term interest rate to conform with the definition of β as a discount factor between adjacent quarters. Finally, to estimate the Euler equations and to construct the user cost of capital for the traditional models, we set the quarterly depreciation rate (δ) to 0.03784 for equipment and 0.01412 for nonresidential structures, the values employed by Bernanke, Bohn, and Reiss (1988).

3. FULL-SAMPLE ESTIMATES OF THE INVESTMENT MODELS

A. The Traditional Models

Table 1 shows full-sample parameter estimates for the traditional investment models described in section 1. Each equation is estimated by ordinary least squares over the period 1955:2 to 1992:4, with all distributed lags allowed to be twelve quarters in length; we estimate these lags without constraints.[9]

Except for the Q model, the coefficients on the distributed lag terms in the equipment equations are strongly significant and of the expected sign, tracing out a hump-shaped distribution with a modal lag of three to four quarters. In contrast, in the Q model, the only significant coefficient on the lags of Q^A is negative, contrary to our expectation. More generally, the Durbin-Watson statistic in each equipment equation is below 0.5, a sign of highly autocorrelated errors. Clearly, these models all fail to capture some persistent determinants of equipment investment.

For nonresidential structures, the performance of the traditional models is even less satisfactory. Although the coefficients in the accelerator and modified neoclassical models have the expected signs, fewer of the lags are significant than was the case for equipment. Moreover, the distributed lags in the neoclassical and Q models are uniformly insignificant, and the Durbin-Watson statistic for each model is even smaller than its counterpart for equipment. The structures models may perform relatively poorly for a simple reason: Roughly half of the structures aggregate consists of public utilities, oil and gas wells, private schools, churches, and hospitals, which have a diverse set of determinants excluded from our models.

B. The Euler Equations

We estimate the Euler equations using the Generalized Method of Moments (GMM) procedure described by Hansen and Singleton (1982). As with the tradition-

9. Our use of OLS departs from the usual method of estimating the traditional investment models with a correction for AR(1) errors. If these models actually captured the dynamics driving investment, the errors would be white noise and an AR(1) correction would not be needed. The AR(1) correction, therefore, should be viewed as a "fix-up" for these models, which should be omitted from a fair horserace with the Euler equations. Given our use of OLS, we calculate standard errors by the Newey-West (1987) procedure that is robust to heteroskedasticity and autocorrelation of unknown form.

TABLE 1

OLS ESTIMATES OF THE TRADITIONAL INVESTMENT MODELS

Variables	Accelerator PDE	Accelerator NRS	Neoclassical PDE	Neoclassical NRS	Q PDE	Q NRS
Constant	4.04**	1.63**	4.18**	1.72**	3.91**	1.96**
	(.072)	(.123)	(.101)	(.104)	(.112)	(.071)
$1/K_{t-1}$	−497.4**	391.5**	−322.8**	475.1**		
	(79.1)	(87.3)	(88.2)	(136.9)		
X_t	.69	−3.31	−.068	−.048	−.569**	−.122
	(2.73)	(4.63)	(.064)	(.032)	(.245)	(.109)
X_{t-1}	10.97**	1.72	.070	−.039	.216	−.007
	(2.04)	(2.98)	(.069)	(.029)	(.217)	(.070)
X_{t-2}	15.97**	3.97	.190**	−.034	.297	.091
	(2.24)	(3.10)	(.069)	(.028)	(.186)	(.060)
X_{t-3}	17.71**	6.32**	.243**	−.015	.209	.111
	(2.64)	(2.77)	(.076)	(.030)	(.173)	(.069)
X_{t-4}	13.44**	5.38	.293**	.003	.188	.043
	(1.92)	(2.92)	(.075)	(.027)	(.159)	(.064)
X_{t-5}	15.11**	7.67**	.276**	.003	.004	.060
	(1.92)	(2.98)	(.082)	(.027)	(.162)	(.064)
X_{t-6}	14.12**	6.97**	.254**	−.005	−.023	.060
	(2.51)	(3.15)	(.064)	(.028)	(.150)	(.064)
X_{t-7}	12.12**	7.84**	.242**	.001	.038	−.023
	(2.39)	(3.08)	(.062)	(.028)	(.160)	(.066)
X_{t-8}	6.07**	4.55	.196**	.009	−.021	.039
	(2.76)	(3.38)	(.071)	(.032)	(.151)	(.077)
X_{t-9}	7.11**	3.43	.210**	.011	.036	−.001
	(3.14)	(2.94)	(.066)	(.029)	(.146)	(.076)
X_{t-10}	11.01**	3.87	.193**	.005	−.099	.027
	(2.34)	(2.95)	(.060)	(.029)	(.153)	(.079)
X_{t-11}	13.12**	5.25	.249**	−.006	−.052	−.195
	(3.45)	(3.76)	(.077)	(.028)	(.204)	(.103)
$\bar{R}^2$	.723	.401	.436	.218	.111	.042
DW	.300	.093	.204	.079	.134	.061

Variables	Modified Neoclassical PDE	Modified Neoclassical NRS	Variables	Modified Neoclassical PDE	Modified Neoclassical NRS
Constant	3.00**	1.95**			
	(.252)	(.265)			
$1/K_{t-1}$	−585.2**	608.2**			
	(93.4)	(195.0)			
X_t	−.066	−.029	W_t	.080	−.020
	(.201)	(.133)		(.271)	(.162)
X_{t-1}	.515**	.139	W_{t-1}	−.418**	−.141
	(.192)	(.102)		(.210)	(.110)
X_{t-2}	.643**	.259	W_{t-2}	−.665**	−.257
	(.245)	(.139)		(.231)	(.146)
X_{t-3}	.735**	.276**	W_{t-3}	−.679**	−.266**
	(.210)	(.114)		(.231)	(.117)
X_{t-4}	.527**	.208**	W_{t-4}	−.566**	−.222**
	(.185)	(.105)		(.185)	(.112)
X_{t-5}	.633**	.278**	W_{t-5}	−.617**	−.291**
	(.192)	(.116)		(.209)	(.115)
X_{t-6}	.624**	.313**	W_{t-6}	−.619**	−.318**
	(.187)	(.117)		(.185)	(.120)
X_{t-7}	.491**	.325**	W_{t-7}	−.524**	−.326**
	(.154)	(.101)		(.178)	(.118)
X_{t-8}	.265	.225	W_{t-8}	−.254	−.240
	(.169)	(.122)		(.174)	(.135)
X_{t-9}	.309	.218	W_{t-9}	−.319	−.224
	(.198)	(.150)		(.202)	(.154)

(*continued*)

TABLE 1 (*Continued*)

Variables	Modified Neoclassical PDE	Modified Neoclassical NRS	Variables	Modified Neoclassical PDE	Modified Neoclassical NRS
X_{t-10}	.559**	.245	W_{t-10}	−.545**	−.269
	(.168)	(.131)		(.190)	(.140)
X_{t-11}	.532**	.256	W_{t-11}	−.508**	−.185
	(.243)	(.173)		(.188)	(.144)
$\bar{R}^2$	.815	.480			
DW	.436	.174			

NOTES: All models were estimated over 1955:2–1992:4, with I_t/K_{t-1} as the dependent variable. To reduce the number of leading zeroes, all reported coefficients and standard errors have been multiplied by 100. The standard errors were calculated with the Newey-West (1987) correction for heteroskedasticity and autocorrelation and are in parentheses.

$$X_{t-i} = \begin{cases} \Delta Y_{t-i}/K_{t-1} & \text{for accelerator model} \\ \Delta(Y/c)_{t-i}/K_{t-1} & \text{for neoclassical model} \\ (1/c)_{t-i-1}Y_{t-i}/K_{t-1} & \text{for modified neoclassical model} \\ Q^A_{t-i} & \text{for } Q \text{ model.} \end{cases}$$

For the modified neoclassical model, $W_{t-i} = (Y/c)_{t-i-1}/K_{t-1}$.
PDE = Producers' durable equipment.
NRS = Nonresidential structures.
DW = Durbin-Watson statistic.
** = significant at the 5 percent level.

al models, we use the Newey-West (1987) method to obtain a covariance matrix for the GMM parameter estimates that is robust to heteroskedasticity and serial correlation.

For GMM to yield consistent parameter estimates, the instruments must be uncorrelated with the error term, ϵ. If the error term were purely an expectational error, rational expectations implies that any variable in the firm's information set during period t would be uncorrelated with ϵ_{t+i} ($i = 1, \ldots, \tau$). In this case, all variables dated $t - 1$ and earlier would be valid instruments, as would endogenous variables chosen in period t. We take a more conservative stance, restricting our instruments to variables dated $t - 2$ and earlier. This decision reflects our concern about the potential for measurement error among the variables in the Euler equations.[10] Because our Euler equations include variables dated as early as $t - 1$ ($IK_t \equiv I_t/K_{t-1}$), ϵ_{t-1} could well include the error component of $t - 1$ dated variables. If so, any endogenous variable would have to be dated $t - 2$ or earlier to be a valid instrument. Accordingly, the instrument set for the basic Euler equation includes a constant and the second and third lags of p^I_t, IK_t, IK^2_t, Y_t/K_{t-1}, and β_t, while the instrument set for the Euler equation with time-to-build also includes the fourth and fifth lags of these variables.[11]

10. In particular, measurement error almost certainly afflicts the price series p^I, given the problem of measuring quality change in capital goods. This error then contaminates the series for real investment and capital stock that BEA constructs from p^I.

11. As a test of robustness, we also estimated the Euler equations with an instrument set that included the first lag of each instrument. The results were not materially different from those reported below. We should note that the exclusion of instruments dated $t - 1$ will not ensure consistent estimates if the measurement errors are autocorrelated. In that case, all lagged endogenous variables will be correlated with the Euler equation's error term. Implicitly, we are assuming that any measurement errors are not strongly autocorrelated.

TABLE 2

GMM ESTIMATES OF INVESTMENT EULER EQUATIONS

	Equipment		Structures	
Parameters	Basic Model (1)	Time-to-Build (2)	Basic Model (3)	Time-to-Build (4)
Production function				
θ	.020	.021	−.193**	−.151**
	(.017)	(.015)	(.048)	(.033)
Adjustment costs				
α_0	−.962**	−1.006**	−4.235**	−3.537**
	(.185)	(.163)	(.822)	(.558)
α_1	1.590	3.965**	−18.334**	−20.735**
	(1.469)	(1.560)	(8.926)	(5.353)
Time-to-build				
ϕ_0		.278**		.174**
		(.090)		(.040)
ϕ_1		.066		.242**
		(.092)		(.044)
ϕ_2		.303**		.270**
		(.114)		(.052)
ϕ_3		.353**		.315**
		(.094)		(.059)
J Statistic	10.33	15.84	7.78	11.46
p-value	.24	.39	.46	.72

NOTES: Basic model was estimated over 1955:2–1992:3, while the time-to-build model was estimated over 1955:2–1991:4. The standard errors are in parentheses and were calculated with the Newey-West (1987) correction for heteroskedasticity and autocorrelation. Estimation of the time-to-build model was done subject to the restriction that $\phi_0 + \phi_1 + \phi_2 + \phi_3 = 1$.

** = significant at the 5 percent level.

To estimate the time-to-build equation, we must specify τ, the length of time between project starts and completions. The previous estimates of structural time-to-build models with quarterly data, Park (1984) and Altug (1989), set τ to be three and four quarters, respectively. These values of τ likely are appropriate for equipment investment, but are too small to encompass some construction projects. However, with a time-to-build lag much longer than four quarters, the large number of free parameters (the ϕs) probably cannot be estimated with any precision. Therefore, we opted to follow the earlier work and set $\tau = 3$, which allows the investment project to be spread over four quarters. We estimate $(\phi_0, \phi_1, \phi_2, \phi_3)$, restricting the ϕ_is to sum to one.

The basic Euler equation without time-to-build is estimated from 1955:2 to 1992:3 (the final period in the sample, 1992:4, provides data for the variables dated $t + 1$). Similarly, the Euler equation with time-to-build is estimated from 1955:2 to 1991:4. The estimates for both equations are shown in Table 2. As can be seen in column 1, the signs of $\hat{\theta}$ and $\hat{\alpha}_1$ in the basic model for equipment are positive, consistent with our expectation. However, neither coefficient is significantly different from zero, and $\hat{\theta}$ is quite small given its interpretation in our model as the share of income accruing to equipment. In addition, the omission of time-to-build from the basic equation appears inconsistent with the data. As shown by the estimates for the time-to-build model in column 2, $\hat{\phi}_0 = 0.278$, indicating that only 27.8 percent of equipment spending occurs during the quarter of the project start; the bulk of investment is estimated to take place two and three quarters after the start. Thus, the Euler equation with time-to-build captures the dynamics of equipment investment better than does the basic equation. Moreover, the addition of time-to-build increases $\hat{\alpha}_1$ by enough to make that coefficient significantly greater than zero. Still, the time-to-build model does not alleviate all the problems with the basic equation, as $\hat{\theta}$ is still very low.

The results for the Euler equations for structures are worse than those for equipment, paralleling our results for the traditional models. As shown in columns 3 and 4, $\hat{\theta}$ and $\hat{\alpha}_1$ are significantly negative for both Euler equations, violating our theoretical priors. Given these nonsensical parameter estimates, the time-to-build model for structures cannot be viewed as a success, even though the ϕs are uniformly positive and significant.

Despite the problems with the Euler equation estimates, the only specification test typically reported for Euler equations—the J statistic— does not reject any of the models at even the 20 percent level.[12] This result illustrates the weakness of the J statistic as an overall test of model specification. Further evidence of the inadequacy of the J statistic can be found in Oliner, Rudebusch, and Sichel (1995), where we show that the estimated parameters of an investment Euler equation appear to be unstable even though the J statistic fails to reject the model.

4. OUT-OF-SAMPLE PERFORMANCE OF THE INVESTMENT MODELS

A. *Generating Out-of-Sample Forecasts*

For each traditional model, the forecast of IK_{t+1} is $\hat{\gamma}_t\mathbf{Z}_{t+1}$, where $\hat{\gamma}_t$ denotes the vector of OLS parameter estimates based on data through period t and $\mathbf{Z}_{t+1}$ denotes the vector of actual values for the explanatory variables in period $t + 1$. We generate a sequence of these one-step-ahead forecasts by extending the sample one period at a time and recalculating $\hat{\gamma}_t\mathbf{Z}_{t+1}$ for each sample. These forecasts are "out-of-sample" in that all coefficients are estimated from data prior to the forecast date. The forecasts, however, are "ex post" because they use the actual values of the explanatory variables at time $t + 1$ in the forecast of investment at time $t + 1$. In real-time forecasting, such values are not available and must be replaced by projections. We analyze ex post forecasts because our interest centers on the adequacy of the investment equations themselves, not the ease of forecasting the explanatory variables.[13]

We apply an analogous procedure to the Euler equations to generate one-step-ahead, ex post forecasts of IK. The specifics of our procedure can be described most easily for the basic Euler equation that omits time-to-build [equation (14)]. First,

12. The J statistic tests the null hypothesis that the instruments are orthogonal to the error term, as required for consistent estimation. This statistic equals the number of observations multiplied by the minimized value of the objective function used in GMM estimation. It is asymptotically distributed χ^2(df), with df equal to the number of instruments minus the number of parameters.

13. However, we did compute ex ante forecasts from the traditional models using univariate autoregressions to generate one-step-ahead forecasts for the explanatory variables. The ex ante forecast errors were almost the same as the ex post errors, as one might expect given the generally small coefficients reported in Table 1 for the contemporaneous explanatory variables.

equation (14) is estimated by GMM using data through period t. Next, we solve the Euler equation for IK_{t+1}. Given the parameter estimates ($\hat{\alpha}_0$, $\hat{\alpha}_1$, $\hat{\theta}$), the assumed value for δ, and the actual values for all variables in the equation other than IK_{t+1}, equation (14) defines a quadratic equation in IK_{t+1}:

$$\left(\frac{\hat{\alpha}_1\beta_{t+1}}{2}\right) IK_{t+1}^2 + (\hat{\alpha}_1(1 - \delta)\beta_{t+1})IK_{t+1} + \hat{W}_{t+1} = 0, \tag{21}$$

where $\hat{W}_{t+1} = (\tilde{\Delta}p_{t+1}^I) + \hat{\alpha}_0((1 - \delta)\beta_{t+1} - 1) - \hat{\alpha}_1 IK_t + \hat{\theta}\left(\beta_{t+}\ \frac{Y_{t+1}}{K_t}\right)$ and we have set the expectational error ϵ_{t+1} to zero. Solving equation (21) yields

$$IK_{t+1} = -(1 - \delta) \pm ((1 - \delta)^2 - 2\hat{W}_{t+1}/(\hat{\alpha}_1\beta_{t+1}))^{1/2}\ , \tag{22}$$

as the forecasting equation for IK_{t+1} . We use the positive branch of equation (22). By extending the sample one period at a time and then repeating this procedure, we obtain the desired sequence of one-step-ahead, ex post forecasts.

The same method yields a forecasting rule for the Euler equation with time-to-build, equation (13). Given GMM estimates of α_0, α_1, θ, and the ϕs, equation (13) can be written as a quadratic in $IK_{t+\tau+1}$:

$$\left(\frac{\hat{\alpha}_1\beta_{t,t+\tau+1}^*}{2}\right) IK_{t+\tau+1}^2 + (\hat{\alpha}_1\hat{\phi}_\tau(1 - \delta)\beta_{t,t+\tau+1}^*)IK_{t+\tau+1} + \hat{W}_{t+\tau+1} = 0\ , \tag{23}$$

$$\text{where } \hat{W}_{t+\tau+1} = \sum_{i=0}^{\tau} \hat{\phi}_i(\tilde{\Delta}p_{t+i+1}^I) + \hat{\alpha}_0 \sum_{i=0}^{\tau} \hat{\phi}_i(\tilde{\Delta}\beta_{t,t+i+1}^*) + \hat{\theta}\left(\beta_{t,t+\tau+1}^*\ \frac{Y_{t+\tau+1}}{K_{t+\tau}}\right)$$

$$+ \hat{\alpha}_1 \sum_{i=0}^{\tau-1} \phi_i(\tilde{\Delta}IK_{t+i+1}) - \hat{\alpha}_1\hat{\phi}_\tau\beta_{t,t+\tau}^* IK_{t+\tau}\ .$$

To make the time subscripts consistent with those in the basic Euler equation, we lag each term in equation (23) by τ periods and then solve (23) for IK_{t+1}:

$$IK_{t+1} = -\hat{\phi}_\tau(1 - \delta) \pm (\hat{\phi}_\tau^2(1 - \delta)^2 - 2\hat{W}_{t+1}/(\hat{\alpha}_1\beta_{t-\tau,t+1}^*))^{1/2}\ . \tag{24}$$

The positive branch of equation (24) generates our ex post forecast of IK_{t+1} for the time-to-build model, based on the actual values for the right-hand side variables and GMM estimates of the parameters computed with data through period t.[14]

14. Equations (22) and (24) are nonlinear functions of estimated parameters. Following Kennedy (1983), we also computed forecasts from equation (22) with a correction for the possible bias from this nonlinearity. The correction made virtually no difference and is not used below. We tried one other sensitivity test for the Euler equation forecasts from equation (22). Rather than using the actual values for right-hand-side variables dated at period $t + 1$, we used the projections of these variables on our instrument set. Strictly speaking, this procedure is more congruent with the rational expectations assumption built into the Euler equations. However, the use of projections rather than actual period $t + 1$ values had no material effect on the Euler equation forecasts.

TABLE 3

SUMMARY STATISTICS FOR EQUIPMENT FORECAST ERRORS

Model	100*RMSE	100*(Forecast Error) Regressed on a Constant		N
		Bias	DW	
Accelerator	.228	−.022 (.51)	.234	116
Neoclassical	.336	.045 (.73)	.233	116
Modified Neoclassical	.227	−.105** (2.92)	.431	116
Q	.396	−.231** (3.75)	.137	116
Basic Euler Equation	9.720	−.078 (.13)	2.431	116
Time-to-Build Euler Eqn.[1]	2.224	.376 (1.18)	.676	113

NOTES: As described in the text, errors are calculated from rolling, one-quarter-ahead forecasts for 1964:1 to 1992:4. All forecasts are out-of-sample, ex post forecasts. Absolute value of t statistics are in parentheses and are calculated from Newey-West (1987) standard errors.

1. Excludes forecasts for three periods for which the model failed to generate a real-valued solution.
RMSE = Root mean square error.
DW = Durbin-Watson statistic.
N = Number of forecast errors.
** = significant at the 5 percent level.

B. *Out-of-Sample Forecast Errors*

Table 3 summarizes the out-of-sample forecast performance of the equipment models over the period 1964:1 through 1992:4. The first column shows the root mean squared error (RMSE) of the one-step-ahead forecasts, multiplied by 100 to remove leading zeroes. The next two columns present statistics derived from a regression of the forecast errors on a constant. Column 2 shows the estimated coefficient from this regression, which equals the mean forecast error, a measure of the forecast's bias. The Durbin-Watson statistic from the regression, shown in the third column, characterizes the extent of first-order autocorrelation in the forecast errors.

Column 1 shows that both Euler equations produce far less accurate forecasts of equipment investment than do the traditional models. The RMSE of the basic Euler equation is roughly twenty-five times larger than that of the worst traditional model, the Q equation. The addition of time-to-build lags markedly improves the forecast performance of the Euler equation, but the RMSE of the time-to-build equation is still well above those of the traditional models. The RMSEs of the traditional models as a group lie in a fairly narrow range, with the accelerator and modified neoclassical models at the low end.

The relatively small RMSEs of the traditional models should not be interpreted as an endorsement of their forecasting ability. The low Durbin-Watson (DW) statistics indicate that the traditional models all make persistent forecast errors. Moreover, the forecasts from the modified neoclassical and Q models have a significant downward bias. The traditional models look good only relative to the performance of the Euler equations.

Table 4 documents the forecast performance of the structures models. As in Table 3, the RMSEs for both Euler equations are many times larger than those of the tradi-

TABLE 4

SUMMARY STATISTICS FOR STRUCTURES FORECAST ERRORS

Model	100*RMSE	100*(Forecast Error) Regressed on a Constant		N
		Bias	DW	
Accelerator	.213	.039 (.94)	.081	116
Neoclassical	.241	.077 (1.73)	.079	116
Modified Neoclassical	.205	.008 (.21)	.117	116
Q	.258	.067 (1.37)	.065	116
Basic Euler Equation	8.735	.525 (.72)	2.331	116
Time-to-Build Euler Eqn.[1]	7.244	1.161 (1.78)	.871	112

NOTES: See Table 3.

1. Excludes forecasts for four periods for which the model failed to generate a real-valued solution.
RMSE = Root mean square error.
DW = Durbin-Watson statistic.
N= Number of forecast errors.

tional models. However, in contrast to the results for equipment, the inclusion of time-to-build lags does not greatly reduce the RMSE of the Euler equation. Another difference from Table 3 is the absence of bias in the forecasts from the traditional models. Still, the forecast errors from these models are highly autocorrelated, with the largest DW statistic at 0.117, suggesting the omission of important explanatory variables.

C. Pairwise Forecast Comparisons

The inability of the Euler equations to forecast out of sample also is evident in pairwise model comparisons. These comparisons are made, following Fair and Shiller (1990), in regressions of the form

$$IK_{t+1} - IK_t = a + \Omega_E(IK^f_{E,t+1} - IK_t) + \Omega_T(IK^f_{T,t+1} - IK_t) + u_t \tag{25}$$

where $IK^f_{E,t+1}$ and $IK^f_{T,t+1}$ are the one-step-ahead forecasts of IK_{t+1} from an Euler equation and a traditional model. Equation (25) regresses the actual change in IK on the predicted change from the two models. If $\Omega_E = 0$, the forecasts from the Euler equation contain no predictive information beyond that in the constant or the traditional model. Conversely, if $\Omega_T = 0$, the forecasts from the traditional model contain no relevant information beyond that in the constant or the Euler equation. If neither model can predict changes in IK, the estimates of both Ω_E and Ω_T should be zero; if both models have predictive power, both Ω_E and Ω_T should be nonzero.

Table 5 displays the pairwise forecast comparisons for the models of equipment investment. The estimates of Ω_E and Ω_T appear in the first two columns, along with t statistics calculated from Newey-West standard errors. The top part of the table

TABLE 5

FAIR-SHILLER REGRESSIONS FOR EQUIPMENT

	Ω_E	Ω_T	Traditional Model	$\bar{R}^2$	N
Basic Euler Equation					
1.	−.0006	.187**	Accelerator	.132	116
	(.74)	(2.64)			
2.	−.0007	.068	Neoclassical	.021	116
	(.94)	(1.49)			
3.	−.0004	.233**	Modified Neoclassical	.175	116
	(.62)	(3.64)			
4.	−.0007	.069	Q	.019	116
	(.88)	(1.45)			
Time-to-Build Euler Equation					
5.	.011**	.200**	Accelerator	.170	113
	(4.04)	(2.80)			
6.	.009**	.065	Neoclassical	.037	113
	(3.03)	(1.40)			
7.	.009**	.235**	Modified Neoclassical	.200	113
	(3.77)	(3.69)			
8.	.008**	.073	Q	.043	113
	(2.37)	(1.48)			

NOTES: Each row reports the OLS estimates of Ω_E and Ω_T from the regression

$$IK_t - IK_{t-1} = a + \Omega_E\,(IK^f_{E,t} - IK_{t-1}) + \Omega_T\,(IK^f_{T,t} - IK_{t-1}) + u_t,$$

estimated over 1964:1–1992:4. $IK^f_{E,t}$ and $IK^f_{T,t}$ are the one-step-ahead forecasts of IK_t from an Euler equation and a traditional model, respectively. Absolute values of t statistics are in parentheses and are calculated from Newey-West (1987) standard errors.

** = significant at the 5 percent level.

compares the basic Euler equation to the traditional models. As shown, the estimates of Ω_E are uniformly insignificant. That is, forecasts from the basic Euler equation have no significant information over and above that provided by the traditional models. In contrast, two of the traditional models—the accelerator and the modified neoclassical models—do have information not conveyed by the Euler equation. However, the relatively low values for $\bar{R}^2$ caution against relying too heavily on any of the models.

The bottom part of Table 5 compares the Euler equation with time-to-build lags to the traditional models. The accelerator and the modified neoclassical models provide information not in the time-to-build Euler equation, similar to the results in the top panel. However, Ω_E is now statistically significant in the comparison with each traditional model. Apparently, the addition of time-to-build lags yields an Euler equation forecast with information not found in the traditional models. Nonetheless, we do not interpret this result as particularly favorable to the Euler equation. First, Ω_E is estimated to be extremely small, suggesting that the Euler equation should get little weight when pooled with the traditional models. Second, despite the significance of Ω_E, the predictive power of the Euler equation is extremely limited. If we omit the traditional model from the Fair-Shiller regression, the $\bar{R}^2$ drops to 0.013. Thus, the Euler equation with time-to-build lags explains only 1.3 percent of the variation in $IK_{t+1} - IK_t$ over the full sample.

Table 6 presents the analogous pairwise comparisons for the models of structures investment. As can be seen, none of the structures models can predict changes in

TABLE 6

FAIR-SHILLER REGRESSIONS FOR STRUCTURES

	Ω_E	Ω_T	Traditional Model	$\bar{R}^2$	N
Basic Euler Equation					
1.	−.0009	.022	Accelerator	.010	116
	(1.62)	(.65)			
2.	−.001	−.0004	Neoclassical	.004	116
	(1.52)	(.01)			
3.	−.0009	.013	Modified Neoclassical	.006	116
	(1.59)	(.42)			
4.	−.001	−.012	Q	.007	116
	(1.46)	(.50)			
Time-to-Build Euler Equation					
5.	.0009	.047	Accelerator	.016	112
	(1.61)	(1.30)			
6.	.0007	.009	Neoclassical	−.009	112
	(1.41)	(.28)			
7.	.0009	.036	Modified Neoclassical	.005	112
	(1.53)	(1.08)			
8.	.0007	−.005	Q	−.009	112
	(1.35)	(.17)			

NOTES: See Table 5.

IK. The estimates of Ω_E and Ω_T are uniformly insignificant at the 5 percent level. Moreover, the values of $\bar{R}^2$ cluster around zero, with the largest value being only 0.016.

5. WHY DO THE EULER EQUATIONS PERFORM SO BADLY?

Several factors could account for the Euler equations' inaccurate forecasts of investment spending. One possibility is that the GMM estimator has poor finite-sample properties [see West and Wilcox (1994) and Fuhrer, Moore, and Schuh (1995) for discussions of this problem in the context of inventory models]. Another possible shortcoming is the use of aggregate data to estimate Euler equations that apply at the firm level. In this section, however, we argue that the Euler equations may well forecast poorly because they impose an invalid dynamic structure on the data.

Consider equation (22), which generates the forecasts of IK_{t+1} for the basic Euler equation, expressed (after some algebra) in the form:

$$IK_{t+1} = -(1 - \delta) + \left[(1 - \delta)^2 + 2\left(\frac{IK_t}{\beta_{t+1}} - \frac{MPK_t - c_t}{\hat{\alpha}_1} \right) \right]^{1/2} , \qquad (26)$$

where $MPK_t = \hat{\theta} Y_{t+1}/K_t$ is the marginal product of capital, and c_t represents the discrete-time version of Jorgenson's user cost of capital.[15] Equation (26) shows that the forecast of IK_{t+1} depends on its own value in period t and on the difference between the marginal product and the cost of capital. The relative importance of

15. Specifically, $c_t = (p_t^I + \alpha_0)(r_{t+1} + \delta) - (1 - \delta)(p_{t+1}^I - p_t^I)$.

each factor reflects the magnitude of marginal adjustment costs, $\hat{\alpha}_1$. If $\hat{\alpha}_1$ is large, then adjusting the rate of investment is quite costly, and IK_{t+1} will deviate relatively little from IK_t, regardless of the difference between MPK_t and c_t. In contrast, as $\hat{\alpha}_1$ approaches zero, $MPK_t - c_t$ becomes the dominant influence on IK_{t+1}. Intuitively, when marginal adjustment costs are very small, the firm quickly adjusts the capital stock to arbitrage away differences between the marginal product of capital and the user cost.[16] Accordingly, the Euler equation will forecast IK_{t+1} to deviate sharply from IK_t whenever $(MPK_t - c_t)/\hat{\alpha}_1$ is large. Given the smoothness of the actual data for IK, the forecasts in such cases will be inaccurate.

The previous section showed that adding time-to-build lags does not remedy the problems with the basic Euler equation, and equation (26) provides the key for understanding this result. In particular, the version of equation (26) for the time-to-build model would replace IK_t and $MPK_t - c_t$ with four-quarter moving averages of these variables that run from period $t - 3$ to period t. Thus, whenever $\hat{\alpha}_1$ is small or a wide gap exists between MPK and c for several quarters, the time-to-build model will forecast big changes in investment. In 1986, for example, the time-to-build model for equipment produced terrible forecasts. The collapse in oil prices that year sharply reduced the price deflator for aggregate output relative to the deflator for equipment alone, driving up real equipment prices (p^I). The resulting capital gain for owners of equipment lowered the user cost, c_t, relative to MPK_t. Accordingly, the time-to-build Euler equation expected equipment outlays to be accelerated in order to take advantage of the low user cost, implying a dramatic decline in IK_{t+1} from its level in period t. In reality, no such intertemporal shift occurred.

As a general matter, the actual series for IK does not display the high degree of time shifting expected by either Euler equation in response to changes in relative prices and interest rates. This problem could well reflect some unrealistic assumptions that underlie both Euler equations. First, the equations embed the "putty-putty" technology of the original neoclassical model. That is, the Euler equations do not distinguish between already-installed capital and capital still to be purchased. These equations expect the capital-output ratio for the entire installed capital stock to adjust to a change in relative prices or interest rates. Such an adjustment, even if done slowly, could induce a large shift in investment outlays. In contrast, a putty-clay model would not allow firms to alter the capital intensity of their existing production facilities.

In addition, the Euler equations assume that investment is fully reversible. Under the more reasonable assumption of irreversibility, Pindyck (1991) and others have shown that investment spending will adjust sluggishly to price changes that generate increased uncertainty about the economic environment.

Finally, the maintained assumption of convex adjustment costs—which generates the investment dynamics in our Euler equations—may be unfounded. Although

16. Note that IK_{t+1} is *negatively* related to $MPK_t - c_t$. That is, $MPK_t > c_t$ implies a low value of IK_{t+1} relative to IK_t. The intuition is simply that the firm shifts investment from period $t + 1$ to period t to capture the profits from the high marginal product of capital.

convex adjustment costs are a convenient assumption, the case for convexity is weak, especially when the adjustments costs are internal to the firm. Convexity implies that the installation of new capital goods should be progressively less costly when dragged out over longer and longer periods. There is no inherent reason why this should be so, a point made originally by Rothschild (1971) and forcefully restated by Nickell (1978), but ignored in most empirical models of investment.

6. CONCLUSION

This paper extends earlier "horse race" comparisons of empirical investment models by adding two Euler equations to the usual stable of traditional—but largely nonstructural—models and by focusing on out-of-sample performance. The basic Euler equation used in the comparisons is a "canonical" Euler equation representative of those found in the applied investment literature. In addition, we use a richer Euler equation with time-to-build lags. Our results indicate that the forecast performance of both Euler equations is substantially worse than that of the traditional models. Although the time-to-build equation performs slightly better than the basic Euler equation, both Euler equations produce forecasts of investment spending that are much too volatile.

Our results have the following implications. First, and most important, the inability of the Euler equations to forecast investment spending even one quarter ahead suggests that these models are misspecified.[17] Investment Euler equations based on simple adjustment cost functions have become a fixture in applied work, but researchers should not assume that these equations are valid structural models. We argued that better models of investment might be provided by Euler equations that embed irreversibility or a putty-clay technology, in order to produce more sluggish adjustments in investment.

Second, none of the models we evaluated could forecast investment in nonresidential structures. This aggregate has a very diverse set of components, and no single model is likely to capture the determinants for all these types of structures. It would be interesting to know whether the traditional models or Euler equations can forecast investment for a single component of the aggregate, such as industrial or commercial buildings.

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