A Nonparametric Investigation of Duration Dependence in the American Business Cycle

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Does the termination probability of a business expansion or contraction increase with age? This question may be formally addressed by analyzing the nature of duration dependence in aggregate economic activity. Our null hypothesis is that there is no duration dependence, which we test via intentionally nonparametric procedures. We also argue that a common notion of business cycle periodicity can be usefully interpreted in terms of whole-cycle duration dependence. We find some evidence for duration dependence in whole cycles and in prewar expansions, but little evidence elsewhere.

I. Introduction

Several authors have recently modeled the business cycle as the outcome of a Markov process that switches between two discrete states, with one of the states representing expansions and the other representing recessions. However, very different specifications have been

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adopted for the transition probability matrix governing the movement of the economy between these two states. For example, Neftci (1982) assumed that the transition probabilities were duration dependent; in particular, he assumed that the longer the economy remained in one state, the more likely it was to change to the other.[1] In contrast, Hamilton (1989) assumed that the state transition probabilities were duration independent so that, for example, after a long expansion (i.e., a long time in the expansion state), the economy was no more likely to switch to the recession state than after a short expansion.[2]

To resolve the question of the duration dependence of expansions and contractions, we investigate the nature of the probability process that generates their lengths. In addition, we consider the evidence for duration dependence in the lengths of whole cycles measured from peak to peak and from trough to trough. Whole-cycle duration dependence is obviously related to the question of half-cycle duration dependence, but it can also be interpreted in terms of a weak definition of stochastic periodicity, namely, that business cycle lengths tend to cluster around a certain duration. We argue that this notion of periodicity was implicit in an earlier literature on business cycles. For example, the classical "8-year" business cycle was distinguished as a cycle by its tendency to endure 8 years.

Of interest, of course, is the *significance* of the tendency of business fluctuations to maintain a fixed cyclical length. Early on, Irving Fisher (1925) argued that business cycles had no such tendency, but that instead they resembled "Monte Carlo cycles," the phantom cycles of luck perceived by gamblers at a casino. Similarly, to a casual observer of a repeated coin toss, runs of consecutive heads or tails may appear more likely to end as they grow longer, but the termination probability of a run actually remains constant. As Fisher would argue, one may tabulate the number of consecutive heads in repeated trials and find the average length of these runs, but there is no intrinsic clustering of run lengths, or periodicity, in the process. It is precisely this interpretation of weak business cycle periodicity that we shall test as our null hypothesis.

In Section II, we explore more fully the notion of duration dependence in a macroeconomic context. In Section III, we provide a weak definition of periodicity that will be useful in interpreting the duration dependence of whole cycles. Section IV describes our empirical methodology, which employs nonparametric tests for duration de-

[1] This view has been expressed often in the popular press, e.g., with the suggestion that a very long expansion is unstable and is unusually likely to end.

[2] This is also the assumption of Diebold and Rudebusch (1989*b*).

pendence. These tests are based on the conformity of the lengths of half cycles and whole cycles to the exponential distribution, which corresponds to an absence of duration dependence. Empirical results are presented in Section V, and Section VI concludes with an interpretation in the light of recent developments in macroeconomics.

II. Macroeconomic Duration Dependence

A large statistical and econometric literature has addressed the interpretation of duration data.[3] A basic element of this analysis is the hazard function, denoted here as $\lambda(\tau)$, which is the conditional probability that a process will end after a duration of length τ, given that it has not terminated earlier. For example, microeconomic data indicate that lengths of employment for individuals exhibit a decreasing hazard function ($d\lambda(\tau)/d\tau < 0$) or negative duration dependence; that is, the longer a job is held, the less likely it is to be lost. This section presents some aspects of duration analysis that are relevant for macroeconomics.

Two examples of hazard functions are shown in figure 1. The constant hazard function, $\lambda_1(\tau) = \lambda$ (dashed line), reflects a termination probability with no duration dependence. The linearly increasing hazard function, $\lambda_2(\tau) = \gamma\tau$ (solid line), reflects a termination probability with positive duration dependence, so that termination probability increases with time. The question of the appropriate specification of a Markov model of the business cycle can be reduced to determining whether expansions and contractions are governed by a constant hazard, as assumed by Hamilton (1989), or by a nonconstant hazard such as $\lambda_2(\tau)$, as assumed by Neftci (1982).[4]

A given hazard function, $\lambda(\tau)$, provides a complete characterization of the unconditional density of durations, $f(\tau)$, since

$$f(\tau) = \lambda(\tau) \exp\left[-\int_0^{\tau} \lambda(u)du\right]. \qquad (1)$$

Figure 2 displays the duration densities associated with the hazard functions given in figure 1. The constant hazard implies an exponential density of durations (dashed line),[5]

$$f_1(\tau) = \lambda \exp(-\lambda\tau), \quad \tau \geq 0. \qquad (2)$$

[3] This literature is well surveyed in Kiefer (1988).

[4] As a related issue, Neftci (1984) investigates whether the hazard rates of expansions and contractions are the same, i.e., whether the state transition matrix and hence business cycles are symmetric. In contrast to his earlier work, Neftci performs the analysis under an assumption of time-invariant transition probabilities.

[5] In discrete time, the corresponding probability distribution is geometric, $f(\tau) = (1 - \lambda)^{\tau-1}\lambda$, $\tau = 1, 2, 3, \ldots$, which has the obvious coin toss interpretation of Fisher.

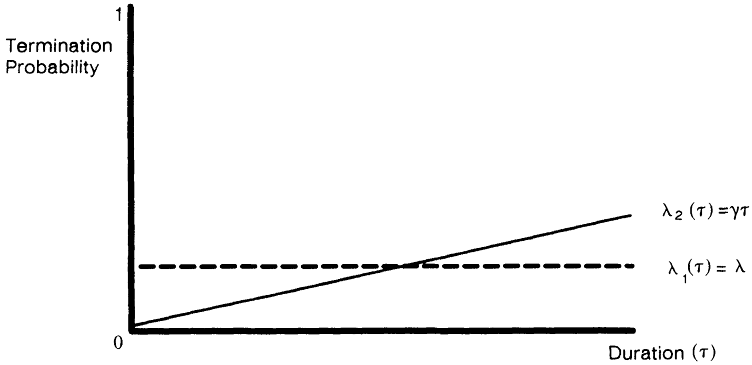

FIG. 1.—Increasing and constant hazard functions

Thus given a constant probability λ of termination, the density of durations is monotonically declining. Alternatively, the linearly upward-sloping hazard implies a particular nonexponential density of durations (solid line),

$$f_2(\tau) = \gamma\tau \exp\left(-\frac{\gamma}{2}\tau^2\right), \quad \tau \geq 0. \tag{3}$$

This density is nonmonotonic and unimodal, and there is a clear concentration of probability mass around the modal value.

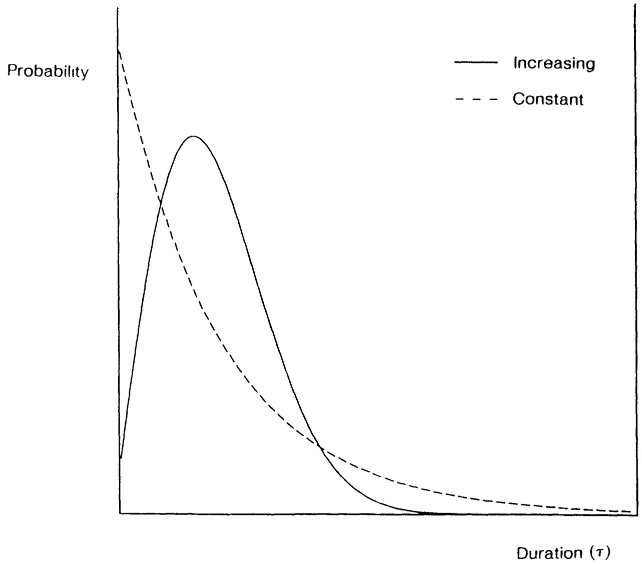

FIG. 2.—Duration distributions associated with increasing and constant hazards

The specific distribution of durations corresponding to a nonconstant hazard will of course depend on that hazard's particular form. In general, however, the probability mass associated with a hazard displaying positive duration dependence is more concentrated around its mean than that associated with the exponential distribution of the same mean.[6] This is an implication of the turning point probability's rising with duration. Consider, for example, the increasing hazard, $\lambda_2(\tau) = \gamma\tau$, which implies a duration density with mean $E(\tau) = (\pi/2\gamma)^{1/2}$ and variance $\text{var}(\tau) = (4 - \pi)/2\gamma$. Note that $d\lambda(\tau)/d\tau$ is positive and increasing in γ, while $\text{var}(\tau)$ is decreasing in γ. That is, as the amount of positive duration dependence increases, the variance of the durations decreases. In addition, the exponential density with an identical mean has a larger variance since the exponential density with mean duration $(\pi/2\gamma)^{1/2}$ has variance $\pi/2\gamma$, which is of course greater than $(4 - \pi)/2\gamma$ for all positive γ.

To summarize, a constant hazard implies an exponential distribution of durations. Thus an exponential distribution of historical lengths of expansions and contractions is precisely the null hypothesis implicit in Fisher (1925) and Hamilton (1989), and it is the one that we shall test below. Furthermore, the positive duration dependence of an increasing hazard induces duration "clustering" around the mean duration, relative to the constant-hazard case. As we describe in the next section, for durations of whole cycles, this clustering has a natural interpretation.

III. Business Cycle Periodicity

In this section, whole-cycle positive duration dependence is related to a weak definition of periodicity, an interpretation that provides intuitive content to the former and empirical content to the latter. To both motivate and clarify our discussion, we shall elucidate several different forms of periodicity, including deterministic and stochastic and strong and weak.

We shall say that a variable X_t displays *deterministic strong periodicity* of period T if $X_{t+T} = X_t$, for all t.[7] This type of periodicity is found in many early macroeconomic models, such as the multiplier-accelerator

[6] This general proposition can be proved, as suggested to us by Martin Wells, by noting the strict concavity of the log survivor function, $\log[1 - F(\tau)]$, when $\lambda(\tau)$ is strictly increasing. Marshall and Olkin (1979, p. 494) show that this concavity implies that the rth moments about the origin, μ_r, are concave in logs when normalized by r factorial ($r!$). In particular, $\log(\mu_1) > ½ \log(\mu_0) + ½ \log(\mu_2/2)$. After rearrangement, this implies that $\text{var}(\tau)$ is less than $[E(\tau)]^2$, which is equal to the variance of the exponential distribution with mean $E(\tau)$.

[7] This definition and the ones that follow abstract from considerations of growth; we also disregard trivial cases such as a constant $X_t = k$.

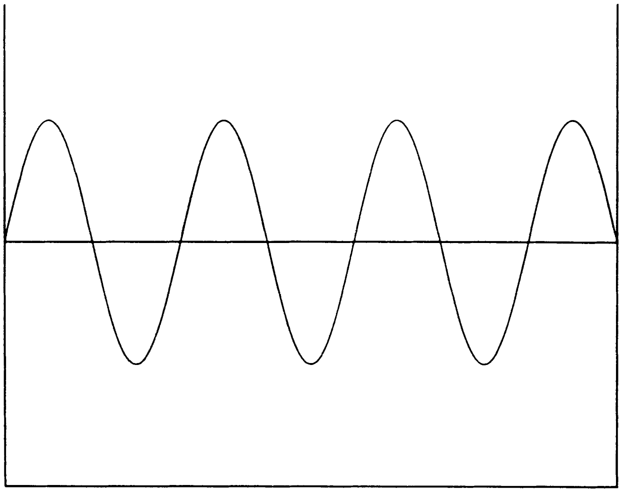

FIG. 3.—Deterministic strong periodicity

and inventory systems of Samuelson (1939) and Metzler (1947). Samuelson's well-known analysis, for example, uses a multiplier-accelerator system to derive a deterministic second-order difference equation for aggregate output. Over a certain range of parameters, this equation produces stable deterministic cycles with a constant period of the type shown in figure 3.

A stochastic framework provides a more realistic basis for analysis of periodicity in economics. The definition of *stochastic strong periodicity* of period T is a straightforward generalization that replaces the equality of X_t and X_{t+T} with a high correlation between these values for all t. Such periodicity has a more precise frequency domain definition as a peak in the spectral density at the frequency corresponding to period T. Frisch (1933) demonstrated that a structural propagation mechanism can convert uncorrelated stochastic impulses into cyclical output with stochastic strong periodicity. This idea of a stochastic, periodic cycle obtained from a perturbed macroeconomic system was the foundation for large-scale macroeconometric models (see, e.g., Klein 1983). However, there has been little empirical support for stochastic strong periodicity in economic fluctuations. Perhaps the most influential evidence against such periodicity is provided by the spectra of macroeconomic variables, which are typically monotonically declining from low to high frequencies (except at seasonal frequencies) with little power concentration at business cycle frequencies (e.g., Granger 1966; Sargent 1987, chap. 11).[8]

[8] This evidence should be interpreted with caution, however, given the small samples

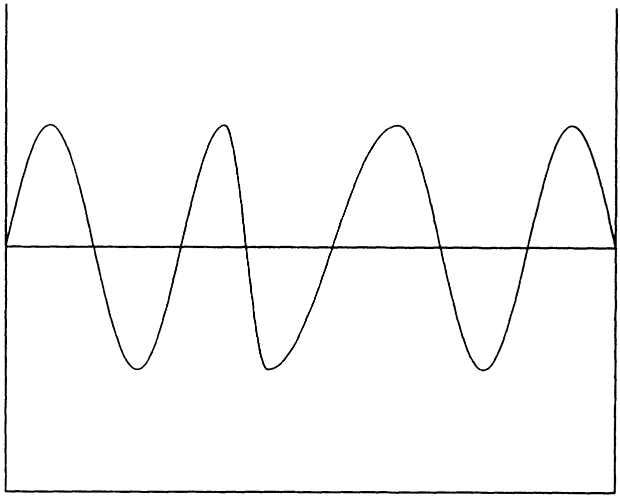

FIG. 4.—Deterministic peak-to-peak weak periodicity

We shall attempt to assess the evidence for a weaker form of periodicity. The essential feature of a strongly periodic process is the close relationship between X_t and X_{t+T} for all t. For the irregular cycles of business activity, weaker forms of periodicity, which depend on periodic repetition for only certain t, are useful. For example, we define *deterministic peak-to-peak weak periodicity* (of period T) to exist for a series if for every t such that X_t is a peak in the series, X_{t+T} is also a peak.[9] This is shown in figure 4 with a series that has uniformly spaced cyclical peaks but is not periodic at every point in the cycle as in figure 3. In particular, note that this series does not exhibit deterministic *trough-to-trough* weak periodicity, which is exhibited when a trough at time t is always followed by a trough at time $t + T$.[10]

involved and the sensitivity of the results to various types of trend adjustment. Furthermore, spectral methods are intrinsically linear and are not compatible with the Markov framework of Neftci (1982) and Hamilton (1989) (see also Neftci 1986; Diebold and Rudebusch 1989*b*).

[9] This definition can be formalized with a function, $TP(\cdot)$, that signals turning points. Specifically, if $Y_t = TP(X_t)$, then Y_t is a sequence that is always zero except at a peak in X_t, when $Y_t = 1$, and at a trough in X_t, when $Y_t = -1$. The series X_t displays deterministic peak-to-peak weak periodicity if, for each t such that $Y_t = 1$, $Y_{t+T} = 1$.

[10] Clearly, strong periodicity implies weak periodicity but not conversely; however, the two definitions of periodicity can be closely linked by a time deformation. Stock (1987) argues that macroeconomic variables appear to evolve on an economic time scale that may speed up or slow down relative to the observed calendar time scale. In such a setting, a cyclical process that is strongly periodic in economic time would be distorted by the nonlinear time deformation into a nonperiodic process in calendar time. However, if the speeding up and slowing down of economic time relative to calendar time averaged out over the cycle, the process would still display weak periodicity in the

The concept of weak periodicity can be extended to a stochastic framework. A series displays *stochastic* peak-to-peak weak periodicity (of period T) if for every X_t that is a peak in the series, $X_{t+\tau}$ is also a peak, where τ is a random variable with mean T and variance σ^2.[11] Stochastic peak-to-peak weak periodicity implies that there is a tight distribution of observed peak-to-peak cycle durations (τ) around the mean period; that is, σ^2 is small. Deterministic peak-to-peak periodicity emerges, of course, when $\sigma^2 = 0$. More generally, however, a natural metric with which to evaluate the size of σ^2, and hence the extent of periodicity, is provided by the exponential distribution. Recall from the last section the close relationship between positive duration dependence and duration clustering relative to an exponential distribution. In particular, if the durations of cycles from peak to peak are clustered around a period of 4 years, then a 2-year-old cycle is less likely to end (i.e., more likely to survive 2 more years) and a 6-year-old cycle is more likely to end (i.e., less likely to survive even longer than 4 years) than a 4-year-old cycle. Thus, for periodic cycles, the probability of a peak is increasing with the length of the ongoing cycle. Nonperiodic cycles, on the other hand, have no particular interval after which they are more likely to end; their turning points are not positively related to the age of the cycle. In this sense, the exponential distribution provides a metric for the extent of periodicity; it allows one to ask whether the distribution of actual business cycle durations is more closely clustered than would be expected from a constant hazard probability model with the same mean duration.

The stochastic weak form of periodicity, defined in terms of a clustering tendency of intervals between turning points, has been used implicitly in many previous discussions of business fluctuations. For example, Matthews (1959, p. 216), in a chapter on business cycle periodicity, implicitly adopts this definition when describing the path of British investment: "Apart from the minor wobbles in the curve around 1877 and 1902, the durations of the cycles measured from trough to trough are 6, 8, 10, 5 years; measured from peak to peak they are 9, 7, 10, 7 years. This is not precisely a seven to ten-year cycle, but it is as near to it as anyone could reasonably expect." The data he presents are suggestive of a clustering of cycle lengths, that is, weak periodicity.[12] We shall examine more rigorously the empirical distri-

constancy of peak-to-peak or trough-to-trough durations. Indeed, fig. 4 is generated by applying precisely such a time deformation to fig. 3.

[11] The stochastic form of weak trough-to-trough periodicity is similarly straightforward.

[12] For other examples, see Adelman and Adelman (1959, p. 614) (who note approvingly the equivalence of peak-to-peak and trough-to-trough durations in the Klein-Goldberger model and in historical cycles), Zarnowitz (1985, pp. 525–26), and Britton

butions of durations of whole cycles and half cycles with procedures detailed in the next section.

IV. Nonparametric Tests for Duration Dependence

We use nonparametric methods to directly test observed durations for conformity to the exponential distribution. Our analysis is intentionally nonparametric since we do not estimate and test a particular hazard model. The imposition of incorrect parametric forms can distort the available departures from the null hypothesis, and it is now well known that incorrect parameterizations of the hazard function can lead to severely misleading inferences (see, e.g., Heckman and Singer 1984).

A description of our testing methodology first requires discussion of the data. The lengths of expansions, contractions, and whole cycles are derived from business cycle turning dates since 1854, as designated by the National Bureau of Economic Research (NBER). These durations (in months) are given in table 1 and provide the raw data for our analysis.[13] By definition, a cycle is designated in the NBER methodology only if it has achieved a certain maturity. Burns and Mitchell (1946, pp. 57–58) describe this criterion: "We do not recognize a rise and fall as a specific cycle unless its duration is at least fifteen months, whether measured from peak to peak or trough to trough. Fluctuations lasting less than two years are scrutinized with special care." Forty years later, Moore and Zarnowitz (1986), in a survey of the NBER methodology, reaffirm this maturity criterion. They indicate that full cycles of less than 1 year in duration and contractions of less than 6 months would be very unlikely to qualify for selection.

Previous examinations of macroeconomic duration dependence, including McCulloch (1975), Savin (1977), and de Leeuw (1987), also have recognized this maturity criterion. However, these earlier stud-

(1986, p. 3). The last of these, which is devoted exclusively to an examination of business cycle periodicity, states that "this 'central tendency' [of cyclical durations] . . . is another way of describing the phenomenon with which the present study is concerned."

[13] In our samples that include postwar expansions, there is a right-censoring problem associated with the current expansion. We have assumed that this last duration is 80 months instead of its unknown, but longer, true length. This affects the durations of the last expansion, the last peak-to-peak cycle, and, with the additional assumption of a following 9-month contraction, the last trough-to-trough cycle. Since the current expansion is already quite long by historical standards, any additional length would shift the results slightly in the direction of no duration dependence. All our results are robust to varying the length of this final expansion over a wide range.

TABLE 1

NBER BUSINESS CYCLE REFERENCE DATES AND DURATIONS

Trough	Peak	Contractions	Expansions	Trough to Trough	Peak to Peak
December 1854	June 1857	NA	30	NA	NA
December 1858	October 1860	18	22	48	40
June 1861	April 1865	8	**46**	30	**54**
December 1867	June 1869	32	18	**78**	50
December 1870	October 1873	18	34	36	52
March 1879	March 1882	65	36	99	101
May 1885	March 1887	38	22	74	60
April 1888	July 1890	13	27	35	40
May 1891	January 1893	10	20	37	30
June 1894	December 1895	17	18	37	35
June 1897	June 1899	18	24	36	42
December 1900	September 1902	18	21	42	39
August 1904	May 1907	23	33	44	56
June 1908	January 1910	13	19	46	32
January 1912	January 1913	24	12	43	36
December 1914	August 1918	23	**44**	35	**67**
March 1919	January 1920	7	10	**51**	17
July 1921	May 1923	18	22	28	40
July 1924	October 1926	14	27	36	41
November 1927	August 1929	13	21	40	34
March 1933	May 1937	43	50	64	93
June 1938	February 1945	13	**80**	63	**93**
October 1945	November 1948	8	37	**88**	45
October 1949	July 1953	11	**45**	48	**56**
May 1954	August 1957	10	39	**55**	49
April 1958	April 1960	8	24	47	32
February 1961	December 1969	10	**106**	34	**116**
November 1970	November 1973	11	36	**117**	47
March 1975	January 1980	16	58	52	74
July 1980	July 1981	6	12	64	18
November 1982	?	16	80	28	96
?		NA	NA	89	NA

NOTE.—The 80-month duration of the last expansion, the 96-month duration of the last peak-to-peak cycle, and the 89-month duration of the last trough-to-trough cycle are conservative estimates. They assume a peak in July 1989 and, for the last of these, a trough 9 months later. Wartime expansions and cycles containing wartime expansions are boldfaced.

ies are limited in two major respects. First, previous analyses examine only the durations of expansions and contractions, but not whole cycles; thus the evidence provided is incomplete.[14] Second, the earlier work obtains evidence on duration dependence from the goodness of fit of estimated sample histograms to a null constant hazard distribution. The power of these tests has been questioned in small sample sizes (see Sichel 1989), and the results obtained with this method often depend on the arbitrary number of cells used in histogram construction (see the sensitivity analysis of Diebold and Rudebusch [1990]).

We shall apply nonparametric tests that have greater power and do not involve the arbitrary factors involved with histogram construction. Rather than grouping observations into histogram bins and thereby discarding information, these tests compare the observations with their ordered rank. The null hypothesis is

$$H_0: f(\tau) = \lambda \exp[-\lambda(\tau - t_0)], \quad \tau \geq t_0, \lambda \text{ unknown}, t_0 \text{ unknown}. \tag{4}$$

That is, the duration random variable τ has an exponential probability density function, where λ has the earlier interpretation as the constant hazard and t_0 is the unknown minimum possible duration from the NBER maturity criterion, which will differ for expansions, contractions, and whole cycles. Shapiro and Wilk (1972) extended their well-known test for normality to provide a similar test for the exponential null H_0. Renumber the durations in ascending order, so that $x_1 \leq x_2 \leq \ldots \leq x_N$; then

$$W = \frac{(\bar{x} - x_1)^2}{(N - 1)\hat{\sigma}^2}, \tag{5}$$

where $\bar{x} = \sum_{i=1}^{N} x_i/N$ and $\hat{\sigma}^2 = \sum_{i=1}^{N} (x_i - \bar{x})^2/N$. The W statistic is a scaled ratio of the squared difference between the mean and shortest duration to the sample variance. The distribution of W is invariant to the true values of λ and t_0, and its exact finite-sample critical values have been tabulated by Shapiro and Wilk for N ranging from three to 100.

Also relevant to our investigation is a modified W statistic developed by Stephens (1978) for testing exponentiality conditional on an assumed known minimum duration, $t_0 = \gamma$, so that the null hypothesis becomes

$$H_0': f(\tau) = \lambda \exp[-\lambda(\tau - \gamma)], \quad \tau \geq \gamma, \lambda \text{ unknown}, \gamma \text{ known}. \tag{6}$$

[14] Expansion and contraction durations individually could show no duration dependence but be negatively correlated during the cycle so as to induce duration dependence in whole cycles.

Define $A = \Sigma_{i=1}^{N} (x_i - \gamma)$ and $B = \Sigma_{i=1}^{N} (x_i - \gamma)^2$. Then the new statistic, denoted $W(t_0 = \gamma)$, is given by

$$W(t_0 = \gamma) = \frac{A^2}{N[(N + 1)B - A^2]}. \tag{7}$$

The statistic $W(t_0 = \gamma)$ has the same distribution for a sample of size N as the W statistic has for a sample of size $N + 1$, so the same table of finite-sample critical values can be used. Both of these statistics allow for the absence of short durations, but the W statistic incorporates a true but unknown t_0 value into the null hypothesis, while $W(t_0 = \gamma)$ *conditions* on an *assumed* t_0 value. The $W(t_0 = \gamma)$ test is useful given information about the NBER maturity criterion and the likely range of the minimum allowable duration t_0; furthermore, a sensitivity analysis that varies t_0 is readily performed.[15]

Finally, we examine another class of nonparametric tests for the exponential distribution. Consider first the null hypothesis H_0 and define the normalized spacings between the ordered durations as

$$Y_i = (N - i + 1)(x_i - x_{i-1}), \quad i = 2, \ldots, N. \tag{8}$$

A plot of Y_i versus i provides a mirror image of the plot of the hazard function; that is, increasing spacings imply a decreasing hazard function. Thus in a regression of normalized spacings on order, namely, $Y_i = \alpha + \beta i$, the exponential hypothesis implies that $\beta = 0$. Brain and Shapiro (1983) exploit this result to obtain a test statistic for exponentiality, denoted Z. Let $\tilde{i}$ and $\tilde{Y}_i$ denote the "de-meaned" variables $i - (N/2)$ and $Y_i - \overline{Y}$. Then

$$Z = \frac{\sum_{i=1}^{N-1} \tilde{i}\tilde{Y}_{i+1}}{\sum_{i=1}^{N-1} Y_{i+1}\left[\sum_{i=1}^{N-1} \tilde{i}^2/N(N - 1)\right]^{1/2}}. \tag{9}$$

The distribution of the Z statistic is asymptotically $N(0, 1)$, which it quickly approaches even in quite small samples. Furthermore, an assumed known minimum duration $t_0 = \gamma$ also can be conditioned on with the Z statistic to test null hypothesis H_0'. Simply consider γ as an additional observation and include as the first weighted spacing $Y_2 = N(x_1 - \gamma)$ in the calculations in equation (9) (running the itera-

[15] A clear trade-off emerges between W and $W(t_0 = \gamma)$. If the conditioning information employed in the latter is correct, it is expected to have higher power; if it is incorrect, nominal and empirical size will diverge. Since the validity of a chosen t_0 value cannot be ascertained exactly a priori in our application, the W and $W(t_0 = \gamma)$ tests are useful in conjunction.

tion from one to N). The modified test statistic is denoted $Z(t_0 = \gamma)$. Brain and Shapiro also provide an alternative statistic, denoted Z^*, that is intended to be more sensitive to alternative duration distributions associated with nonlinear hazard functions.[16] The statistic Z^* is constructed from a linear regression *and* a quadratic regression of Y_i on order and has an asymptotic chi-squared distribution that appears, from the simulation study in Brain and Shapiro, to be appropriate even in small samples.

A number of Monte Carlo studies have examined the power of the W and Z tests against various alternatives, including the Weibull, chi-squared, half-normal, and lognormal distributions.[17] Overall, the W and Z tests appear to be comparable in their ability to detect departures from exponentiality, with small comparative advantages for one or the other against specific alternative distributions. Both appear to have excellent power in the range of small sample sizes relevant for our analysis.

V. Empirical Results

Besides performing the constant hazard tests on the full samples of expansions, contractions, and peak-to-peak and trough-to-trough cycles, we also examine a variety of subsamples. These include only pre– or post–World War II observations and may exclude wartime expansions and the whole cycles that contain them. The various duration samples investigated are listed in table 2 with their associated sample size, mean duration, and standard error.[18] The variation in the standard error, one measure of dispersion, anticipates some of our later statistical results, which will also account for sample size, mean duration, and minimum duration.

Our study of various subsamples is an attempt to control for possible heterogeneity across cycles. We are interested in duration dependence induced by economic behavior, and the chosen sample should reflect intrinsic macroeconomic forces rather than special factors. That is, the *systematic* mechanism of business cycles, which are prop-

[16] For example, with a hump-shaped hazard function, the slope of the fitted linear regression line, $Y_i = \alpha + \beta i$, may be close to zero. Thus the Z and $Z(t_0 = \gamma)$ statistics, which are based on this slope, may not be sensitive to such alternatives.

[17] Besides power studies in the papers cited above by Shapiro, Wilk, Brain, and Stephens, there are also relevant results in Samanta and Schwarz (1988).

[18] It can be argued that the success of macroeconomics and macroeconomic policy has been the halving of the mean duration of contractions in the postwar period. This point is different from the one Baily (1978) made about diminished postwar amplitudes, which was disputed by Romer (1989) but reaffirmed by Balke and Gordon (1989).

TABLE 2

BUSINESS CYCLE, EXPANSION, AND CONTRACTION SAMPLES

Sample	Sample Size	Mean Duration	Standard Error
Expansions:			
E1. Entire sample	31	34.6	21.8
E2. Entire sample, excluding wars	26	28.9	15.3
E3. Post–WWII	9	48.6	28.9
E4. Post–WWII, excluding wars	7	40.9	22.3
E5. Pre–WWII	21	26.5	10.7
E6. Pre–WWII, excluding wars	19	24.5	9.2
Contractions:			
C1. Entire sample	30	18.1	12.5
C2. Post–WWII	9	10.7	3.4
C3. Pre–WWII	21	21.2	13.6
Peak to peak:			
PP1. Entire sample	30	52.8	24.9
PP2. Entire sample, excluding wars	25	47.9	22.0
PP3. Post–WWII	9	59.2	31.0
PP4. Post–WWII, excluding wars	7	51.6	26.0
PP5. Pre–WWII	20	47.9	20.3
PP6. Pre–WWII, excluding wars	18	46.6	20.9
Trough to trough:			
TT1. Entire sample	31	52.3	22.1
TT2. Entire sample, excluding wars	26	47.4	17.8
TT3. Post–WWII	9	59.0	27.5
TT4. Post–WWII, excluding wars	7	51.3	19.3
TT5. Pre–WWII	21	47.7	18.1
TT6. Pre–WWII, excluding wars	19	45.9	17.6

erly considered a modern phenomenon of market economies, should be distinguished from *accidental* and episodic crises associated with wars, bad harvests, and foreign manipulation of oil prices.[19] Although one can always find circumstances specific to each cycle, to the extent that all business cycles are alike in their essentials, any intrinsic duration dependence should be evident. In the absence of any clear information on the size or direction of the bias associated with large, episodic exogenous shocks, we have some preference for complete samples.[20]

[19] See Burns and Mitchell (1946, chap. 1). For an evaluation of the role that such shocks have played in directing the path of U.S. economic fluctuations, see Blanchard and Watson (1986).

[20] Large, exogenous shocks may bias the evidence for weak economic periodicity in either direction. For example, the coincidence of two oil price shocks in 1974 and 1979 or the existence of a quadrennial political business cycle may spuriously strengthen the evidence. (See Britton [1986, chap. 6] for a discussion.)

TABLE 3

W and $W(t_0 = \gamma)$ Tests for Exponentiality
(p-Values under the Null of No Duration Dependence)

Sample	Statistic			
	$W(t_0 = 8)$	$W(t_0 = 9)$	$W(t_0 = 10)$	W
Expansions:				
E1	.360	.533	.699	.573
E2	.113	.194	.410	.211
E3	.512	.573	.633	.420
E4	.524	.592	.660	.310
E5	<.01	.019	.044	.015
E6	<.01	.018	.043	<.01
	$W(t_0 = 4)$	$W(t_0 = 5)$	$W(t_0 = 6)$	W
Contractions:				
C1	.725	.990	.672	.810
C2	.044	.150	.580	.188
C3	.436	.548	.859	.904
	$W(t_0 = 13)$	$W(t_0 = 15)$	$W(t_0 = 17)$	W
Peak to peak:				
PP1	<.01	.016	.037	.017
PP2	.015	.039	.085	.042
PP3	.351	.467	.581	.250
PP4	.509	.625	.741	.317
PP5	.010	.028	.057	.021
PP6	.040	.079	.151	.073
	$W(t_0 = 13)$	$W(t_0 = 15)$	$W(t_0 = 17)$	W
Trough to trough:				
TT1	<.01	<.01	<.01	.698
TT2	<.01	<.01	<.01	.748
TT3	.136	.182	.280	.735
TT4	.086	.117	.162	.523
TT5	<.01	<.01	.010	.778
TT6	<.01	<.01	.026	.971

Note.—These finite-sample p-values are obtained by linearly interpolating the tables in Shapiro and Wilk (1972) The samples are identified in table 2.

Probability values for the test statistics are given in tables 3 and 4. These p-values represent the likelihood of obtaining the value of the test statistic actually obtained under the null of no duration dependence.[21] Small p-values therefore indicate significant departures from exponentiality. We generally prefer the third column of each table, that is, the $W(t_0 = \gamma)$ and $Z(t_0 = \gamma)$ tests, which assume a minimum

[21] The tests employed require that the observations are independent. In fact the correlations between successive durations in table 1 are quite low and are not statistically significant at even the 20 percent level.

TABLE 4

Z, Z^*, AND $Z(t_0 = \gamma)$ TESTS FOR EXPONENTIALITY
(p-Values under the Null of No Duration Dependence)

SAMPLE	$Z(t_0 = 8)$	$Z(t_0 = 9)$	$Z(t_0 = 10)$	Z	Z^*
Expansions:					
E1	.383	.574	.818	.579	.077
E2	.165	.292	.491	.291	.028
E3	.705	.781	.862	.547	.320
E4	.701	.780	.866	.488	.382
E5	.021	.043	.090	.033	.008
E6	.022	.047	.099	.034	<.005
	$Z(t_0 = 4)$	$Z(t_0 = 5)$	$Z(t_0 = 6)$	Z	Z^*
Contractions:					
C1	.587	.952	.453	.662	.052
C2	.096	.252	.672	.268	.351
C3	.393	.633	.956	.974	.067
	$Z(t_0 = 13)$	$Z(t_0 = 15)$	$Z(t_0 = 17)$	Z	Z^*
Peak to peak:					
PP1	.011	.027	.067	.028	<.005
PP2	.018	.043	.103	.043	<.005
PP3	.535	.653	.792	.406	.357
PP4	.677	.814	.972	.487	.431
PP5	.018	.038	.080	.028	<.005
PP6	.045	.087	.169	.064	<.005
	$Z(t_0 = 13)$	$Z(t_0 = 15)$	$Z(t_0 = 17)$	Z	Z^*
Trough to trough:					
TT1	<.005	<.005	.009	.964	.633
TT2	<.005	<.005	.006	.960	.416
TT3	.294	.371	.467	.921	.502
TT4	.210	.269	.346	.713	.709
TT5	<.005	.008	.018	.943	.162
TT6	.006	.014	.031	.838	.050

NOTE.—The p-values are obtained using the asymptotic distributions of the Z and $Z(t_0 = \gamma)$ statistics, which are $N(0, 1)$, and of the Z^* statistic, which is χ^2 with two degrees of freedom. The samples are identified in table 2.

duration equal to the shortest observed duration (i.e., 17 months for cycles, 10 for expansions, and 6 for contractions). The first two columns in each table check the robustness of the results with smaller t_0 values, while the W, Z, and Z^* columns do not incorporate information regarding the likely range of t_0. We also generally prefer the W statistics over the Z statistics since their exact finite-sample critical values are available.

Consider first the W, Z, and Z^* tests that do not condition on a particular choice of t_0. When the expansion sample is taken as a whole, the case for positive duration dependence appears very slight.[22] Exclusion of wartime expansions leads to a reduced p-value, but we still fail to reject the null of no duration dependence at conventional significance levels. Significant duration dependence is indicated for prewar expansions, especially when wars are excluded, while postwar expansions show no evidence of duration dependence, regardless of whether wars are excluded. There is no evidence for duration dependence in any of the samples of contractions; however, in contrast to expansions, there is more evidence for duration dependence in the postwar period (though not significant at conventional levels) than in the prewar period.[23] It is interesting to note that, while there is generally little evidence of duration dependence in either expansions or contractions, there is significant duration dependence over the entire cycle, measured peak to peak.[24]

The Z test results are in solid agreement with those of the W test. The Z^* test results also accord quite closely, leading us to suspect that most departures from the constant-hazard null hypothesis are monotone.

We now report the results of the $W(t_0 = \gamma)$ and $Z(t_0 = \gamma)$ tests, which make use of conditioning information on t_0. An upper bound (and, in fact, a reasonable choice) for t_0 is the actual shortest observed duration. Thus our preferred t_0 value is 6 months for contractions and 10 months for expansions. For peak-to-peak cycles, the shortest duration is 17 months, which is about the sum of the shortest contraction and expansion lengths. For trough-to-trough cycles, the shortest duration is 28 months; however, with no evidence of a distinction by the NBER in designating the two types of cycles,[25] we prefer a t_0 of 17 months for each type of complete cycle. This conditioning information has one important effect. The results for trough-to-trough cycles now closely match those obtained for peak-to-peak cycles and imply positive duration dependence in most samples.

[22] The nature of the deviation from exponentiality, if any, can be inferred from the sign of the Z statistics, which were negative for all significant or near-significant departures from the null. The sign of the Z statistic is the same as the slope of the regression of the normalized spacings on the order, which is the inverse of the regression of the durations on the order. Thus negative Z statistics are associated with positive duration dependence.

[23] One interpretation of this result is that postwar countercyclical policy has been at least partially successful in terms of increasing duration dependence in contractions; i.e., contractions cluster around the smaller mean.

[24] As will be seen shortly, there is also strong evidence of duration dependence in trough-to-trough cycles, which the W, Z, and Z^* tests fail to detect. This is due to the minimum duration of 28 months for most of the trough-to-trough cycles, which are implicitly used by the W, Z, and Z^* tests as the minimum duration. The tests with lower, more reasonable, minimum durations do detect duration dependence in trough-to-trough cycles.

[25] Recall the Burns and Mitchell statement of Sec. IV.

The results from the $Z(t_0 = \gamma)$ test are very similar to those obtained with the $W(t_0 = \gamma)$ test. The differences between the first three columns of tables 3 and 4 are very slight. Notably, the $Z(t_0 = 17)$ column for whole cycles closely agrees with the $W(t_0 = 17)$ column.

We believe that our results, which for the most part suggest whole-cycle positive duration dependence and half-cycle duration independence, can be fruitfully reconciled. Whole-cycle duration dependence can take several different forms. Clearly, if both halves of the cycle exhibit duration dependence, so will the whole cycle. In addition, duration dependence of just expansions or of just contractions (with no duration dependence for the other half cycle) could generate cyclical duration dependence. Finally, if neither half cycle displays duration dependence but their lengths are negatively correlated, the whole cycle may display duration dependence. If duration dependence and weak periodicity were an important and intrinsic feature of the business cycle, one would expect that one of the forms would predominate over the sample. Our results on half-cycle duration dependence indicate that this is not the case; instead, the significant whole-cycle duration dependence appears to be a mixture of all these possibilities. The slightly significant prewar expansion duration dependence and the almost significant postwar contraction duration dependence coupled with a slight negative correlation between half-cycle durations drive the whole-cycle results.[26] This clearly qualifies our whole-cycle results since it admits the possibility that they are a spurious coincidence of several factors.

VI. Conclusion

Our examination of the complete samples of expansions and contractions uncovered little evidence for duration dependence, which suggests that the maintained assumption of constant Markov transition probabilities in Hamilton (1989) is legitimate. In the postwar sample, our results indicate that this assumption appears to be particularly valid for expansions and perhaps less so for contractions, although the very small size of these samples may impair the power of the tests.

In contrast to our results for expansions and contractions, we have found some indication of duration dependence in whole cycles, although these results must be qualified by the uncertain and varying nature of the duration dependence. However, if durations of cycles are indeed more tightly clustered than those associated with an expo-

[26] Over the whole sample, the correlation between an expansion and the following contraction is $-.21$, and for a contraction with the following expansion it is $-.04$. Neither of these, however, is significant.

nential distribution, then this appears to provide evidence against Fisher's hypothesis of a "Monte Carlo" business cycle. The positive cyclical duration dependence suggests weakly periodic behavior and hence a business cycle that cannot be completely characterized only by examination of *comovements* among macroeconomic aggregates (as in Lucas [1977]). Stochastic weak periodicity, as manifested by positive duration dependence, may be an important feature of American business fluctuations, in addition to the obvious multivariate interactions. Here, however, more research is required to assess the economic significance of any duration dependence rather than just its statistical significance.

By directly examining durations of NBER-designated expansions, contractions, and cycles, we beneficially avoided conditioning on a particular model. However, it will be of interest to ascertain the duration dependence properties of various theoretical macroeconomic models. Models of recent vintage, whether of the new-classical, new-Keynesian, or real business cycle variety, are simple Frischian impulse propagation mechanisms. The nature of the fluctuations implied by such models therefore depends, of course, on the propagation structure of the system and the nature of the impulses driving the system. It is a relatively straightforward exercise to explore the nature of duration dependence in the intertemporal equilibria implied by various economic models, given a filter for identifying turning points. There is, however, little agreement on the appropriate form of such a filter. The judgmental NBER filter, for example, does not have an exact, explicit representation.

Similarly, it will also be of interest to ascertain the duration dependence properties of various statistical models commonly used as reduced-form descriptions of business cycle dynamics. In particular, although we have used the nonlinear Markov switching model to motivate the issues treated in this paper, questions of duration dependence arise naturally in many other contexts as well. Given a definition of turning points, for example, one would like to inquire about the nature of duration dependence associated with various linear and nonlinear, stationary and nonstationary, dynamic statistical models. This is especially interesting in the light of the fact that there is little agreement regarding an appropriate statistical model, whether linear or nonlinear. For example, among linear models, consensus has not yet been reached on the existence and importance of shock persistence associated with unit roots; the relative importance of the permanent and transient components in gross national product has been the subject of considerable debate (see, e.g., Campbell and Mankiw 1987; Cochrane 1988; Diebold and Rudebusch 1989*a*). The notions of business cycle duration dependence introduced here may aid in discrimination among such competing models, via their introduction of a fresh metric for comparing economic models to data.

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