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Scoring the Leading Indicators*

I. Introduction

To economic agents suffering through cycles of prosperity and depression, the prospect of a set of indicators that could provide advance warning of economic fluctuations is tantalizing. Leading cyclical indicators of U.S. aggregate economic activity have had continued popularity in the 50 years since their original development by Wesley Mitchell and Arthur Burns (1938).¹ The use of leading indicators has survived the criticism of “measurement without theory,” first leveled by Koopmans (1947), as well as the rise (and partial decline) of the large-scale structural modeling approach to econometric forecasting. The release of the composite index of leading indicators is trumpeted each month by the popular and financial press, although the interpretation and significance that should be attached to the latest numbers are often unclear. This article provides a rigorous analysis of the predictive ability of the

We evaluate the ability of the composite index of leading indicators to predict business cycle turning points. Formal probability-assessment scoring rules are applied to turning-point probabilities generated from the leading index via a Bayesian sequential probability recursion. These scoring rules enable rigorous and systematic evaluation of leading indicator forecasts. The results are used to assess the merits of forecasting with the composite leading index and to suggest possible improvements in its construction.

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1. Paul Samuelson (1987) recalls an even earlier construction of leading indicators, the Harvard ABC curves, which were popular in the 1920s.

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composite leading index in forecasting business cycle peaks and troughs. In particular, we apply formal probability-assessment scoring rules to the cyclical turning-point probabilities generated from the index of leading indicators via Neftci's (1982) sequential probability recursion.

The next section provides theoretical justification for the use of leading indicators in economic prediction, with a focus on the prediction of cyclical turning points. The informational content of a leading indicator can be evaluated only in the context of a given method of prediction, so various methods of translating leading signals into turning-point predictions are discussed. In Section III, we consider standards for evaluating the predictive performance of the leading indicators through a number of formal probability-assessment scoring rules that naturally complement the sequential probability recursion. The quadratic probability score, the probability-forecast analog of mean-squared prediction error, and additional measures of probability-forecast calibration and resolution are introduced. These scoring rules characterize the forecasting ability of the leading indicators in a number of dimensions.

The fourth and fifth sections contain empirical results. In Section IV, the probability forecasts are calculated, and several methodological innovations are introduced into the sequential probability recursion. These forecasts are scored in Section V. The final section contains concluding remarks and suggests directions for future research.

II. Prediction with Leading Indicators

Leading economic indicators have long been used in the prediction of business cycle peaks and troughs. Many have argued that the composite leading index (CLI) is particularly useful in the analysis of business conditions if attention is placed on its ability to predict an economic event (a turning point), rather than to forecast future values of economic time series. This view contrasts with the regression-based approaches of Auerbach (1982) and Neftci (1979), who examine the predictive power of linear regressions of coincident variables on leading indicators.² It also contrasts with the work of Wecker (1979) and Kling (1987), which involves translating such linear projections into turning-point forecasts. Both have a foundation in the linear regression framework, where a prediction error of a given size carries the same weight, regardless of the point in the cycle at which it occurs. Consequently, a

2. Auerbach (1982), for example, finds that the composite leading index is causally prior to unemployment and industrial production. This contrasts with Neftci (1979), who shows that the individual component indicators, taken one at a time, are less useful for predicting changes in cyclical variables. (Koch and Rasche [1988] report similar results.) This suggests that the benefits of "portfolio diversification" are one motivation for the use of a composite index.

good fit at the turning points can be overwhelmed by a poor fit at the majority of data points between turning points.

In this article, we take an event-oriented, nonregression-based approach, motivated by the belief that the economy behaves differently in the downturn phase than in the upturn phase of the cycle and, in particular, that turning points delineate essential changes in the empirical relations among economic variables. Okun (1960), Hymans (1973), Zarnowitz and Moore (1982), Moore (1983), and others have noted that the turning points of business cycles are special and that the composite index of leading indicators has been constructed so as to maximize the amount of *turning-point* information available.³ Thus, the CLI as currently constructed may not be best suited for prediction and evaluation in the classical minimum mean-squared prediction error framework; the information content of the leading indicators has been focused on the occurrence of an economic event, the business cycle turning point, not on the value of an economic variable. Leading indicator information may be qualitative and event oriented, providing a signal of changes in economic regime.

Early writers on the business cycle paid close attention to the different mechanisms operating at peak and trough (“crisis and revival”) and during expansion and contraction.⁴ One difference is the apparent asymmetry of the business cycle: the long, gradual expansion versus the short, steep contraction, as discussed in Neftci (1984). Recent econometric work in the areas of switching and threshold models, nonlinear dynamical systems, and nonlinear filtering is in the same spirit.⁵ Given different behavior of the macroeconomy during expansions and contractions, the dynamic optimization problems faced by agents imply the desirability of predicting turning points. In such a switching economy, there is an advantage in forecasting both the expected future value of an economic variable and its future probability structure, as delineated by turning points. For example, Neftci (1982) considers the case where Y_t is a stochastic variable representing major macroaggregates such as employment or production, and Y_t has two different probability distribution functions $G^u(Y_t)$ and $G^d(Y_t)$. The first represents probabilities associated with Y_t during the upswing or expansion regime; the second represents probabilities during the downswing or contraction regime. A turning point, peak or trough, is defined

3. For example, in presenting a major redefinitional revision of the CLI, Zarnowitz and Boschen (1975) describe the most important selection criterion for choosing components of the index in terms of turning points: “The consistency of cyclical timing is crucially important for the principal use of the indicators: timely recognition (ideally, for the leading series, reasonably successful prediction) of business cycle turning points. Hence timing is accorded the highest weight.”

4. See, e.g., Haberler (1937) and Schumpeter (1939).

5. See, e.g., Tong (1983); Hamilton (1987); and Brock and Sayers (1988).

as the point in time when the probability distribution changes from $G^u(Y_t)$ to $G^d(Y_t)$ or vice versa. The prediction problem can then be separated into forecasting the period when the distributions switch and then obtaining forecasts of the future values of Y_t .

In general, when the relationships among the major macroaggregates change with the phase of the business cycle, a separate prediction of turning points will be useful. For example, a businessman may project product sales based on a projection of aggregate gross national product (GNP). The relationship between sales and GNP may be very different when the economy is expanding from when it is contracting; thus, it is fruitful to predict not only the future values of GNP but also the regime. Moreover, regime-specific business-cycle behavior may be institutionalized, as are the federal budget deficit targets mandated by the Gramm-Rudman-Hollings legislation, which, by law, are suspended automatically during a recession.

Given a separate role for the prediction of turning points, there is now a place for a "leading indicator," a series that portends the downturn or upturn. In forecasting situations, a leading indicator is only as good as the rule used to interpret its movements, that is, the procedure that maps leading-indicator changes into turning-point predictions. The determination of rules that yield "early warnings" while minimizing "false alarms" is analogous to the construction of statistical tests with good properties in terms of type I and type II errors. The classic example of a turning-point filter associated with the CLI is the three-consecutive-declines rule for signaling a downturn. (See, e.g., Vaccara and Zarnowitz 1977.) More sophisticated rules have been studied by Okun (1960), Hymans (1973), and Zarnowitz and Moore (1982).

However, as described in Neftci (1982), a class of real-time sequential-analytic leading-indicator prediction rules can be rigorously formulated for the switching economy. Label the coincident indicator Y_t , which, as above, switches probability distribution at turning points. A leading indicator X_t also switches distribution (a turning point) but with some lead time over the turning point in Y_t . The forecaster tries to recognize the change in the probability distribution of X_t with enough lead time to fruitfully predict the turning point in Y_t . Let Z_X be an integer-valued random variable that represents the time-index date of the first period after the turning point in X_t . For example, in the prediction of a downturn,

$$\begin{aligned} X_t &\sim F^u(X_t), & 1 \leq t < Z_X, \\ X_t &\sim F^d(X_t), & Z_X \leq t, \end{aligned}$$

where F^u and F^d are the respective upturn and downturn distributions. Time-sequential observations on the leading indicator are received, so at time t , there are $(t + 1)$ observations denoted $\bar{x}_t = (x_0, x_1, \dots, x_t)$.

At time t , we calculate a probability for the event $Z_X \leq t$, that is, that by time t a turning point in X has occurred.

The probability of $Z_X \leq t$ after observing the data \bar{x}_t at time t can be decomposed by Bayes's formula:

$$P(Z_X \leq t | \bar{x}_t) = \frac{P(\bar{x}_t | Z_X \leq t)P(Z_X \leq t)}{P(\bar{x}_t)}.$$

Define $\Pi_t = P(Z_X \leq t | \bar{x}_t)$ as the posterior probability of a turning point given the data available. As shown in Appendix A, we obtain a very convenient recursive formula for the posterior probability prediction of a downturn:

$$\begin{aligned} \Pi_t = & [\Pi_{t-1} + \Gamma_t^u \cdot (1 - \Pi_{t-1})]f^d(x_t | \bar{x}_{t-1}) / \{[\Pi_{t-1} + \Gamma_t^u \\ & \cdot (1 - \Pi_{t-1})]f^d(x_t | \bar{x}_{t-1}) + (1 - \Pi_{t-1})f^u(x_t | \bar{x}_{t-1})(1 - \Gamma_t^u)\}, \end{aligned}$$

where $\Gamma_t^u = P(Z_X = t | Z_X \geq t)$, the probability of a turning point peak in period t given that one has not already occurred, and f^u and f^d are the probability densities of the latest (t th) observation if it came from, respectively, an upturn or downturn regime (in X_t) and conditional upon previous observations.⁶ (To predict the probability of an upturn or trough, exchange f^u with f^d and use the transition probability Γ_t^d , the probability of a trough in t given a continuing contraction.) With this formula, the probability Π_t can be calculated sequentially by using the previous probability Π_{t-1} , a "prior" probability that $Z = t$ based solely on the distribution of previous turning points, and the likelihoods of the most recent observation x_t based on the distribution of X_t in upswings and downswings. Given Π_t , a probability forecast about the value of Z_X , the forecaster can then relate this to Z_Y , the occurrence of a turning point in Y_t . In practice, the probability of a turning point in X_t is mapped into the probability of an "imminent" turning point in Y_t over a fixed horizon.

The precise application of this formula to the problem of forecasting the business cycle is described in Section IV. The sequential turning-point probabilities supplied by the above formula have been applied by Neftci (1982) and Palash and Redeki (1985) in a sequential-analytic optimal stopping-time framework, where a decision maker at each point in time faces the choice of whether to signal the occurrence of a recession or not to signal one and wait for more observations.⁷ We instead examine these probabilities directly, treating them as forecasts in and of themselves, much like meteorological probability of precipita-

6. The conditional probability Γ_t^u is equivalent to the transition probability (expansion regime to contraction regime) in a Markov formulation.

7. In the optimal stopping-time framework, a turning point is deemed "imminent" if $\Pi_t > \Pi_t^*$, a critical value chosen to yield a small probability type I error, at which point "sampling" stops. We make direct use of all probabilities and never stop sampling.

tion forecasts. The next section describes statistics that directly assess the informational content of such probability forecasts generated with the CLI.

III. Evaluation of Probability Forecasts

On an ex post basis, one may simply examine turning points in the composite leading index and tabulate their lead times relative to reference cycle turning points. However, recognition of CLI turning points may be much more difficult in real time, so that truly objective evaluation requires ex ante real-time filtering rules, such as the sequential probability recursion (SPR), for detecting turning points in the CLI. In other words, while good ex post turning-point lead-time performance is a necessary characteristic of an ex ante useful CLI, it is not sufficient.

An evaluation of the CLI can only be conducted within a given methodology for translating CLI movements into forecasts; thus, any such evaluation is conditional on the methodology adopted. A systematic evaluation of the probability forecasts generated via the SPR has not yet appeared in the literature; here we perform such an evaluation by using a variety of techniques available for evaluating probability forecasts. We evaluate leading-indicator turning-point forecasts on a number of attributes, including accuracy, calibration, and resolution. Precise definitions of statistics measuring these attributes are given below, though not in their most general form where separate probabilities of many possible outcomes must be considered (see Diebold 1988). For our purposes, the universe consists of only two (mutually exclusive) events, the occurrence or nonoccurrence of a turning point, so the formulae simplify considerably.

Our first attribute for forecast evaluation is *accuracy*, which refers to the closeness, on average, of predicted probabilities and observed realizations, as measured by a zero-one dummy variable. Suppose we have time series of T probability forecasts $\{P_t\}_{t=1}^T$, where P_t is the probability of the occurrence of a turning point at date t (or, more generally, over a specific horizon H beyond date t). Similarly, let $\{R_t\}_{t=1}^T$ be the corresponding time series of realizations; R_t equals one if a turning point occurs in period t (or over the horizon H) and equals zero otherwise. The probability-forecast analog of mean squared error is Brier's (1950) quadratic probability score:⁸

$$\text{QPS} = 1/T \sum_{t=1}^T 2 (P_t - R_t)^2.$$

8. It is worth noting that use of such a quadratic loss function may not be appropriate in all contexts. In particular, loss, if symmetric, need not grow as the square of the error. It is also not clear that symmetric loss is appropriate. For example, if a forecaster is

The QPS ranges from 0 to 2, with a score of 0 corresponding to perfect accuracy. Moreover, the QPS has the desirable property of being strictly proper, meaning that it achieves a strict minimum under truthful revelation of probabilities by the forecaster. In addition, it is the unique proper scoring rule that is a function only of the discrepancy between realizations and assessed probabilities, as shown by Winkler (1969).

We also consider another strictly proper accuracy-scoring rule, the log probability score (LPS), given by

$$\text{LPS} = -1/T \sum_{t=1}^T \left[(1 - R_t) \ln(1 - P_t) + R_t \ln(P_t) \right].$$

The LPS ranges from 0 to ∞ , with a score of 0 corresponding to perfect accuracy. The LPS depends exclusively on the probability forecast of the event that actually occurred, assigning as a score the log of the assessed probability. In the two-event universe of this article, the LPS is a fully general scoring rule, because the probability forecast of a turning point (P_t) implicitly determines the probability forecast of a nonturning point ($1 - P_t$). The loss function associated with LPS differs from that corresponding to QPS, as large mistakes are penalized more heavily under LPS.

The *calibration* of a probability forecast refers to closeness of forecast probabilities and observed relative frequencies. Overall forecast calibration is measured by global squared bias: $\text{GSB} = 2 (\bar{P} - \bar{R})^2$, where $\bar{P} = 1/T \sum_{t=1}^T P_t$ and $\bar{R} = 1/T \sum_{t=1}^T R_t$.

Clearly, $\text{GSB} \in [0, 2]$, with $\text{GSB} = 0$ corresponding to perfect global calibration, which occurs when the average probability forecast equals the average realization. One can also consider the calibration of sets of probability forecasts. Partition the series of probability forecasts into $j = 1, \dots, J$ cells with T^j forecasts in each cell ($\sum T^j = T$). Then, within-cell forecast calibration is measured by local squared bias:

$$\text{LSB} = 1/T \sum_{j=1}^J 2 T^j (\bar{P}^j - \bar{R}^j)^2,$$

where \bar{P}^j is the within-cell average probability and \bar{R}^j is the average realization of turning points associated with these forecasts. Like GSB, $\text{LSB} \in [0, 2]$, and zero corresponds to perfect local calibration. While $\text{LSB} = 0$ implies $\text{GSB} = 0$, the converse is not true.

Resolution (RES) measures the extent to which different forecasts

penalized more heavily for “missing a call” (i.e., making a type II error) than for “signaling a false alarm” (i.e., making a type I error), then the appropriate loss function is asymmetric.

are in fact followed by different realizations. Formally,

$$\text{RES} = 1/T \sum_{j=1}^J 2 T^j (\bar{R}^j - \bar{R})^2.$$

Thus, we measure resolution as a weighted average of squared deviations of cell realization means from the grand mean, where the weights are given by the number of probability forecasts falling within each cell. RES is simply a weighted variance of the \bar{R}^j values (thus $\text{RES} \geq 0$), and high resolution indicates that discriminating predictive information is available. To see this, consider the case in which all cell means are equal to the grand mean. Then the forecast has no resolution at all ($\text{RES} = 0$), the cell means being constant (at \bar{R}) regardless of the predicted probability values. Murphy (1973) has established the important decomposition:

$$\text{QPS} = \text{QPS}_{\text{const}} + \text{LSB} - \text{RES},$$

where $\text{QPS}_{\text{const}}$ is the QPS of the constant probability forecast \bar{R} .

We thus have three attributes on which to evaluate probability forecasts (given with their related scoring measures): accuracy (QPS, LPS), calibration (GSB, LSB), and resolution (RES). In Section V, we shall use these measures to actually “score” the composite index of leading indicators in the prediction of cyclical turning points.

IV. Empirical Analysis—Generation of Probabilities

To use the recursive formula of Section II to construct probability forecasts of business cycle turning points, we must obtain the transition probabilities $\{\Gamma_t^u\}$ and $\{\Gamma_t^d\}$, as well as the densities f^d and f^u , and an initial condition Π_0 . The sequence of peak and trough conditional transition probabilities depends on the stochastic structure generating regime lengths. Neftci (1982) calculates transition probabilities (expansion to contraction regime) that increase with the age of the regime by using the relative frequencies of observed CLI turning points. It is not clear, however, that the probability of a turning point should increase as current regime continues, for example, that a long expansion is more likely to end than a short one. In other work, we have presented evidence that the expansions and contractions in the American business cycle, particularly in the postwar period, are not characterized by duration dependence; thus, the probability of a turning point is roughly independent of the age of the regime.⁹ These results have been replicated for turning points in the CLI. Consequently, we provide sequen-

9. See Diebold and Rudebusch (1988*b*, 1988*c*); see also McCulloch (1975).

tial probability forecasts using these time-invariant transition probabilities (i.e., $\Gamma_t^u = \Gamma^u$ and $\Gamma_t^d = \Gamma^d$).

The sequential probability recursion also requires the probability density of the leading series given that the stochastic generating structure is expansion ($f^u(\cdot)$) and given that it is contraction ($f^d(\cdot)$). The leading series used in this paper is the percent change in the composite index of leading indicators. The division of the leading series into regimes depends on the underlying classification of economic activity. We have followed the *Business Conditions Digest* in denoting peaks and troughs of the CLI that correspond to the National Bureau of Economic Research (NBER) business cycle, and both chronologies are given in Appendix B. After grouping the leading indicator observations into two classes corresponding to upswing and downswing regimes, we estimate the relevant densities f^u and f^d . We have experimented with several procedures to obtain these densities, and following Neftci (1982), all are based on the assumption of a simple probability structure of X_t of the form

$$X_t = \begin{cases} \alpha^u + \epsilon_t^u & \text{in expansions,} \\ \alpha^d + \epsilon_t^d & \text{in contractions,} \end{cases}$$

where α^u and α^d are fixed and ϵ_t^u and ϵ_t^d are independently and identically distributed (iid) zero-mean random variables with variances σ_u^2 and σ_d^2 , respectively. This extremely simple model may be viewed as providing an approximation to a switching density, conditional on regime.¹⁰

To estimate f^u and f^d , we fit a simple normal density function to observations in each regime. The procedure is easily replicated and provides a good approximation to the underlying data. A number of completely nonparametric density estimates, such as those of Terrel and Scott (1985), were also considered with no substantive effect on the results.

The final element in the sequential probability recursion is last period's posterior probability of a turning point. There are two corrections made to this probability in practice. First, at the start of a new regime, a start-up probability of zero is used as the previous period's probability. Also, as is clear from the formula, if the posterior probability reaches one at any point, it will force all remaining probability forecasts to be one in the regime. Thus, we put an upper bound of .95

10. Extensions to time-varying intraregime conditional densities are straightforward conceptually but quite tedious in practice, as can be inferred from App. A. In particular, convenient analytic recursions, such as the SPR given above, are not available in the more general case. To the extent that superior approximations could be obtained from more sophisticated nonlinear models, our results provide a *lower bound* on the predictive performance of the CLI.

on the previous posteriori probability as it enters the recursive probability formula.

An example of probabilities obtained from the SPR is given in figure 1. While a rigorous evaluation of these probabilities will be provided in the next section, it is helpful to first give some general discussion. In figure 1, the probabilities are obtained by using fitted normal densities $f^u(x_t)$ and $f^d(x_t)$ and a uniform transition prior: $\Gamma_t^u = .02$ during expansions and $\Gamma_t^d = .10$ during contractions, for all t . The dates of NBER peaks and troughs are denoted by vertical dashed lines. The turning point probabilities start in December 1948 and continue to August 1986. Those preceding a peak (trough) refer to the probability of the beginning of a recession (expansion). (In the figure, the probabilities are reported for 5 months past the turning point, but these late probabilities will not be scored.) The probabilities in figure 1 provide a signal for the onset of every recession except the very sudden downturn of 1981; however, the lead times of these signals are variable and, in particular, too long in 1957. Four false signals of recession are given: two major ones in 1951 and 1966, two minor ones in 1962 and 1984. It is instructive to compare these SPR probability signals with those produced by alternative rules, such as three consecutive declines in the CLI. The plus signs (+) in figure 1 denote the third month in each string of three consecutive declines in the CLI. There is, in general, a correspondence between the triple decline signal and high probabilities of recession. The probability measure, however, gives more information, and it is *sustained* information of a quantitative type.¹¹ The probability measure, for example, clearly distinguishes the triple CLI decline in 1962 as a false signal within 2 months while sustaining the recession signal in 1969. In addition, this quantitative information is often given with a greater lead time as in 1979, where the probability of imminent recession reaches 80 percent in the month before the third decline. The probabilities in figure 1 preceding a trough give the likelihood of an imminent upturn. There is no simple rule of thumb that signals troughs available for comparison, so we will defer further evaluation to Section V.

Before scoring these probabilities, several qualifications and caveats should be noted. The probability estimates, while sequential and real time in spirit, are not completely *ex ante*, out-of-sample forecasts. First, they make use of some quantities, in particular the f^u and f^d densities, which are estimated over the entire sample. Second, the numbers used for the CLI are of a final revised form, whereas, in real-time forecasting, only preliminary and partially revised data are available. In addition, the components of the CLI are often changed and

11. The three-consecutive-declines rule ignores the *magnitude* of the fall in the CLI; it does not distinguish, e.g., between three 1% declines and three 0.1% declines.

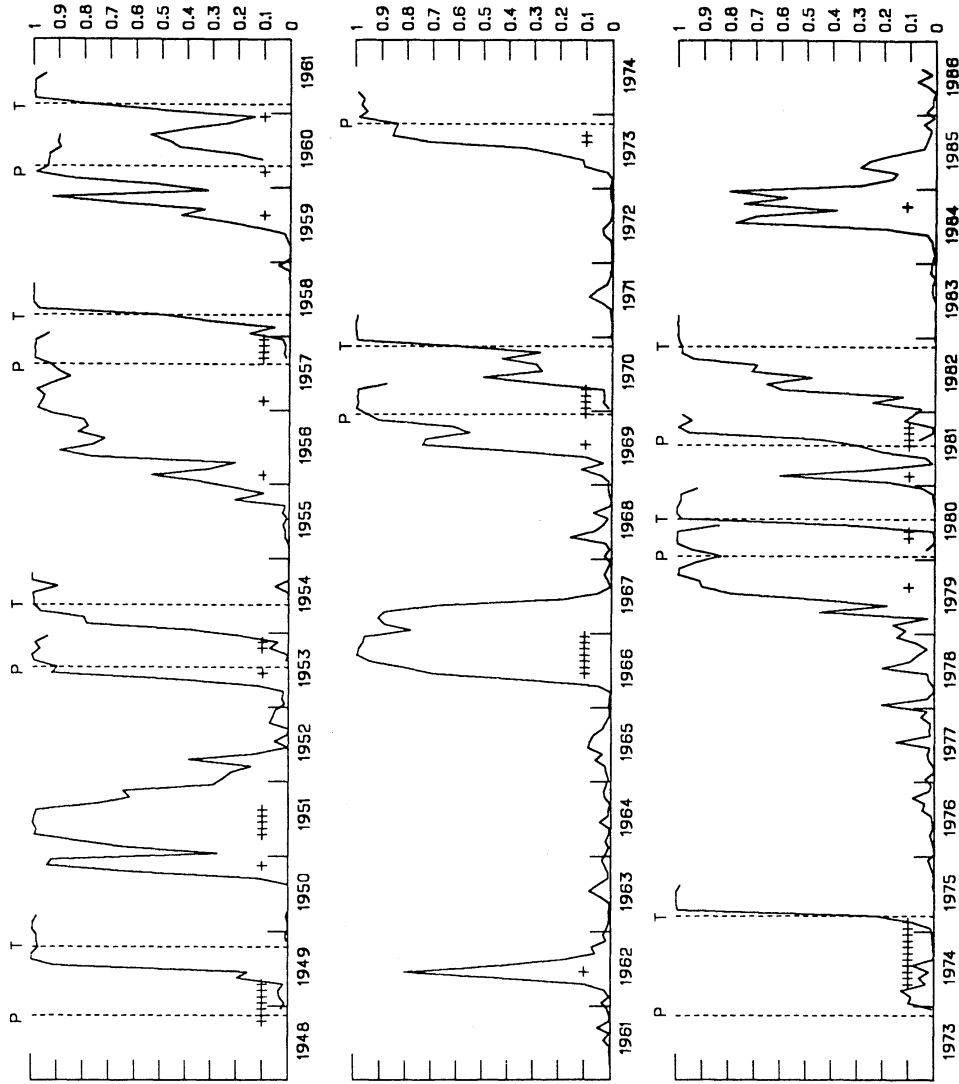


Fig. 1.—Sequential probabilities of a turning point

reweighted *ex post* to improve performance over the sample. Thus, for instance, the 1971 CLI data that we use was not available in 1971 but is the most recent (1987) formulation of the CLI reconstructed for 1971. Completely *ex ante* forecasts would involve both a rolling sample construction of densities based on previous data and the use of the preliminary and first-revision original construction CLI data that was available in real time.¹²

While the above two qualifications would tend to induce an overestimation of the performance of the CLI, there are also a number of lines of reasoning that suggest an underestimation of the performance of the CLI based on an incorrect assessment of false signals. First, note that we evaluate and score the CLI on how well it predicts NBER business-cycle turning points. Insofar as the CLI portends mere economic slowdowns and growth-cycle turning points and insofar as a policymaker is interested in an early warning of such near recessions, then the scoring of the CLI probability forecasts only with respect to business cycles may be misleading. (See J. Shiskin's comments following Hymans [1973].) A second and more subtle point is that if the leading indicators have been used in the formation of effective countercyclical policy, then they will be evaluated as less effective than they really are. For example, in 1966 the CLI signaled a forthcoming recession, and if policymakers took that signal and avoided a recession, then the signal, although a proper one, would be labeled, *ex post*, as false.

With these qualifications of potential over- and underassessment of the CLI noted, we now provide a detailed analysis of the turning-point forecasts.

V. Empirical Analysis—Scoring of Probabilities

In this section, we analyze the sequential turning-point probability forecasts generated in the previous section. A comparison of the scores of SPR forecasts with other probability forecasts, including constant-probability forecasts and variants of the CLI three-consecutive-declines rule, allows us to provide a joint characterization of the usefulness of the SPR and the information content of the CLI. Tables 1–6 present scoring attributes for probability forecasts of peaks and troughs generated by this variety of methods. Given differences in the dynamics of upswings and downswings, we might expect differences in predictive performance of the composite index of leading indicators when forecasting peaks versus troughs. This suggests that the actual

12. Hymans (1973) examines, in the context of a simple forecasting rule, the effect of using original CLI data in forecasting and finds a negligible effect, while Zarnowitz and Moore (1982) find a somewhat larger effect. For further discussion of the properties of revisions in the leading indicators, see Diebold and Rudebusch (1988*a*). For a completely *ex ante* analysis, see Diebold and Rudebusch (1988*b*).

scoring calculations (rather than just choice of the transition probabilities Γ^u and Γ^d in the generation stage) should be performed separately on probabilities generated in expansions and contractions.

Let us first evaluate the forecasts in terms of accuracy. Tables 1 and 2 present the Quadratic Probability Score (QPS) and the Log Probability Score (LPS) for each forecasting technique. While both statistics measure accuracy, the implicit loss functions differ. The forecasting methods include a no-change, NAIVE forecast, which amounts to a constant zero probability forecast, $P_t = 0$, of a downturn or upturn. This is the probability forecast analog of a random walk (in this case, $\text{QPS} = 2\bar{R}$). More generally, one can search in the zero-one interval for the number that is the most accurate as a probability prediction of turning points. Such optimal, CONSTANT probability forecasts are of the form $P_t = \kappa^u$ during expansions and $P_t = \kappa^d$ during contractions, where the constants are chosen to minimize QPS or LPS. In the second row of table 1, for example, at a forecast horizon of 5 months, a 10% probability forecast of a downturn ($\kappa^u = .10$, given in parentheses below the score) is the most accurate constant-probability forecast. These optimal constant-probability forecasts are a natural first choice for the prior probabilities used in the generation of posterior turning-point probabilities via the SPR (i.e., $\Gamma^u = \kappa^u$ and $\Gamma^d = \kappa^d$ in the rows labeled SPR). An alternative constant prior could be chosen by searching the zero-one interval for the constant prior that provides the most accurate probabilities generated via the SPR. Such probabilities (scored in rows SPR*) are the best that can be generated from the CLI by the SPR. Finally, two variants on the “three-consecutive-declines” theme for the prediction of downturns were evaluated. The simplest rule of three, denoted 3CD, produces probability forecasts of zero or one, depending on whether the most recent three observations have been negative. A number of methods were used, in an attempt to enhance these probability forecasts, with some success. In particular, a linear decay method is also scored (3CDa) where generated forecasts of unity are followed by .8, .6, .4, .2, 0.0 (unless, of course, three more consecutive declines occur, at which time the probability immediately returns to 1.0).

A number of interesting features emerge from tables 1 and 2. All methods score best at short horizons (1–3 months), and predictive performance deteriorates with horizon.¹³ In addition, as alluded to earlier, the predictive performance of all techniques differs sharply between expansions and contractions. In particular, troughs are harder to

13. This is consistent with the results of other studies on average lead time. While the performance of trough forecasts appears to improve at very long horizons (9–12 months), this is merely a manifestation of the fact that contractions typically are short, so that, given a long enough horizon, one can always obtain accurate, though useless, trough predictions by forecasting a turning point with probability one.

TABLE 1 QPS as a Function of Horizon, Various Forecasting Methods

Method	Forecast Horizon					
	1	3	5	7	9	13
Prediction of peaks:						
NAIVE	.04	.12	.19	.27	.35	.52
CONSTANT	.04	.11	.18	.24	.29	.38
(κ^u)	(.02)	(.06)	(.10)	(.14)	(.18)	(.26)
SPR	.29	.35	.38	.38	.41	.49
(Γ^u)	(.02)	(.06)	(.10)	(.14)	(.18)	(.26)
SPR*	.04	.12	.19	.23	.28	.39
(Γ^u)	(10^{-7})	(.0001)	(.001)	(.005)	(.01)	(.03)
3CD	.19	.23	.26	.28	.34	.50
3CD _a	.24	.23	.23	.24	.29	.45
Prediction of troughs:						
NAIVE	.18	.55	.91	1.25	1.55	1.88
CONSTANT	.17	.40	.50	.47	.35	.13
(κ^d)	(.09)	(.27)	(.45)	(.63)	(.78)	(.93)
SPR	.27	.41	.45	.48	.42	.17
(Γ^d)	(.09)	(.27)	(.45)	(.63)	(.78)	(.93)
SPR*	.15	.29	.39	.43	.37	.15
(Γ^d)	(.005)	(.07)	(.23)	(.39)	(.95)	(.97)

NOTE.—The CONSTANT probability forecast is a constant (given in parentheses) that minimizes the QPS. The SPR probabilities use this constant probability as a prior (Γ^u or Γ^d , given in parentheses). The SPR* probabilities are generated with the prior transition probabilities (given in parentheses) that minimize the QPS. The forecast horizon is given in months, and the scoring sample is from December 1948 to December 1986.

TABLE 2 LPS as a Function of Horizon, Various Forecasting Methods

Method	Forecast Horizon					
	1	3	5	7	9	13
Prediction of peaks:						
NAIVE	.27	.80	1.34	1.89	2.44	3.53
CONSTANT	.10	.22	.32	.40	.47	.57
(κ^u)	(.02)	(.06)	(.10)	(.14)	(.18)	(.26)
SPR	.49	.60	.66	.66	.69	.85
(Γ^u)	(.02)	(.06)	(.10)	(.14)	(.18)	(.26)
SPR*	.13	.24	.35	.40	.48	.70
(Γ^u)	(10^{-5})	(.0003)	(.002)	(.005)	(.02)	(.05)
3CD	1.33	1.56	1.80	1.89	2.37	3.45
3CD _a	1.26	1.27	1.30	1.38	1.73	2.81
Prediction of troughs:						
NAIVE	1.26	3.77	6.28	8.63	10.68	12.87
CONSTANT	.30	.59	.69	.66	.54	.25
(κ^d)	(.09)	(.27)	(.45)	(.63)	(.78)	(.93)
SPR	.56	.64	.73	.75	.64	.41
(Γ^d)	(.09)	(.27)	(.45)	(.63)	(.78)	(.93)
SPR*	.35	.48	.66	.70	.63	.41
(Γ^d)	(.005)	(.09)	(.26)	(.50)	(.72)	(.94)

NOTE.—The CONSTANT probability forecast is a constant (given in parentheses) that minimizes the LPS. The SPR probabilities use this constant probability as a prior (Γ^u or Γ^d , given in parentheses). The SPR* probabilities are generated with the prior transition probabilities (given in parentheses) that minimize the LPS. The forecast horizon is given in months, and the scoring sample is from December 1948 to December 1986.

predict than are peaks. At a 3-month horizon, for example, SPR* has a QPS of .12 for peak prediction and a QPS of .29 for trough prediction.

These accuracy scores shed light on two important additional issues: first, the performance of the SPR forecasts relative to other rules for interpreting movements in the CLI and, second, the performance of the CLI-based forecasts relative to benchmark naive and constant-probability forecasts. To address the first issue, compare the SPR* scores to those obtained from applying the three-consecutive-decline rules (3CD and 3CDa).¹⁴ These rules are in general outperformed by the SPR at all horizons; note, in particular, the poor log-probability scores obtained by the simple rules. With regard to the second issue, the information content of the CLI, we compare the SPR, SPR*, NAIVE, and CONSTANT forecast rows. While SPR* performs much better than the naive, no-change forecast, its comparative advantage relative to the optimal constant probability forecast (CONSTANT) is less pronounced. Relative performance differs significantly over expansions and contractions: in the prediction of troughs, SPR* is generally superior to CONSTANT, while the two methods produce similar results in the prediction of peaks. This holds true regardless of whether the QPS or LPS loss function is used.

We can characterize the performance of the probability forecasts in greater detail by examining other scores. The extent of bias in the forecasts for various horizons is given in a global sense (GSB) over upswing and downswing observations in table 3. All forecasts except the naive, no-change forecast and the SPR forecast are well calibrated, that is, correct on average. The direction of the bias of the NAIVE forecast is, of course, one of underprediction of returning-point probabilities. For the SPR (where $\Gamma^u = \kappa^u$, $\Gamma^d = \kappa^d$), however, the bias is one of overprediction of the probabilities of turning points.¹⁵

The weighted average of the biases associated with particular forecasts (e.g., the 25% probability forecast of recession compared with the associated realized relative frequency), or local squared bias (LSB), is given in table 4.¹⁶ Again, the overprediction bias of the SPR is evident. The local calibration of the SPR* forecasts is elucidated further by examining the relationship between the individual probability forecasts and resulting relative frequency of realizations (table 5). The

14. An evaluation in which the predictive power of monetary and financial variables is explored and lead times are the sole evaluation criterion is provided in Palash and Radecki (1985) and favors the SPR.

15. This is clear since the unbiased SPR* requires lower priors and hence involves lower posteriors. Since these optimal priors, which are natural ones to use, are unbiased, the overprediction or false alarm bias of the posterior SPR probabilities reflects either deficiencies in the CLI or in our application of the forecasting methodology.

16. Following standard practice, our continuous probability forecasts were discretized by mapping [0, .1) into .05, [.1, .2) into .15, etc. This LSB discretization is responsible for the slight differences in GSB and LSB for the NAIVE and CONSTANT forecasts.

TABLE 3 GSB as a Function of Horizon, Various Forecasting Methods

Method	Forecast Horizon					
	1	3	5	7	9	13
Prediction of peaks:						
NAIVE	.00	.01	.02	.04	.06	.13
CONSTANT	.00	.00	.00	.00	.00	.00
	(.02)	(.06)	(.10)	(.14)	(.18)	(.26)
SPR	.08	.11	.12	.12	.11	.10
	(.02)	(.06)	(.10)	(.14)	(.18)	(.26)
SPR*	.00	.00	.00	.00	.00	.00
	(10^{-7})	(.0001)	(.001)	(.005)	(.01)	(.03)
3CD	.01	.00	.00	.00	.02	.05
3CD _a	.03	.02	.00	.00	.00	.02
Prediction of troughs:						
NAIVE	.02	.15	.41	.78	1.19	1.74
CONSTANT	.00	.00	.00	.00	.00	.00
	(.09)	(.27)	(.45)	(.63)	(.78)	(.93)
SPR	.07	.06	.02	.00	.00	.00
	(.09)	(.27)	(.45)	(.63)	(.78)	(.93)
SPR*	.00	.00	.00	.02	.05	.00
	(.005)	(.07)	(.23)	(.39)	(.95)	(.97)

NOTE.—The CONSTANT probability forecast is a constant (given in parentheses) that minimizes the QPS. The SPR probabilities use this constant probability as a prior (Γ^u or Γ^d , given in parentheses). The SPR* probabilities are generated with the prior transition probabilities (given in parentheses) that minimizes the QPS. The forecast horizon is given in months, and the scoring sample is from December 1948 to December 1986.

TABLE 4 LSB as a Function of Horizon, Various Forecasting Methods

Method	Forecast Horizon					
	1	3	5	7	9	13
Prediction of peaks:						
NAIVE	.00	.00	.00	.02	.03	.08
CONSTANT	.00	.00	.00	.00	.00	.00
	(.02)	(.06)	(.10)	(.14)	(.18)	(.26)
SPR	.25	.26	.23	.20	.19	.19
	(.02)	(.06)	(.10)	(.14)	(.18)	(.26)
SPR*	.00	.03	.05	.05	.06	.06
	(10^{-7})	(.0001)	(.001)	(.005)	(.01)	(.03)
3CD	.14	.10	.07	.05	.05	.09
3CD _a	.20	.14	.08	.05	.04	.07
Prediction of troughs:						
NAIVE	.00	.10	.33	.66	4.04	1.56
CONSTANT	.00	.00	.00	.00	.00	.00
	(.09)	(.27)	(.45)	(.63)	(.78)	(.93)
SPR	.14	.13	.12	.19	.13	.07
	(.09)	(.27)	(.45)	(.63)	(.78)	(.93)
SPR*	.04	.03	.09	.12	.06	.04
	(.005)	(.07)	(.23)	(.39)	(.95)	(.97)

NOTE.—The CONSTANT probability forecast is a constant (given in parentheses) that minimizes the QPS. The SPR probabilities use this constant probability as a prior (Γ^u or Γ^d , given in parentheses). The SPR* probabilities are generated with the prior transition probabilities (given in parentheses) that minimize the QPS. The forecast horizon is given in months, and the scoring sample is from December 1948 to December 1986.

TABLE 5 Reliability Analysis of SPR* Forecasts

P_j	\bar{R}_j	$P_j - \bar{R}_j$	No. of Forecasts	% of Forecasts
Prediction of peaks:				
.05	.11	.06	201	57
.15	.29	.14	28	8
.25	.25	.00	14	4
.35	.40	.05	15	4
.45	.46	.01	6	2
.55	.57	.02	9	3
.65	.22	.33	9	3
.75	.50	.25	10	3
.85	.45	.40	19	5
.95	.62	.33	42	12
Prediction of troughs:				
.05	.24	.19	29	33
.15	.14	.01	7	8
.25	.14	.11	7	8
.35	.40	.05	5	6
.45	.40	.05	5	6
.55	.75	.20	4	5
.65	1.00	.35	4	5
.75	.50	.25	6	7
.85	.33	.52	6	7
.95	1.00	.05	15	17

NOTE.—For expansions $H = 12$ and $\Gamma = .03$; for contractions $H = 5$ and $\Gamma = .23$.

actual frequency distribution of probability forecast values is also show. Taken together, the entries in the table enable us to examine the joint distribution of forecasts and realizations, as factored into the distribution of realizations conditional on forecasts and the marginal distribution of the forecasts. For illustrative purposes, we constructed table 5 using a horizon of 12 for expansions and a horizon of 6 for contractions, with optimal transition probabilities of .03 and .23, respectively. The feature of note (for both expansions and contractions) is the local bias associated with both very small probability forecasts and very large probability forecasts, which illustrates the problem of false alarms and missed calls. The mid-range probability forecasts, however, display little systematic bias.

The resolution (RES) scores, given in table 6, provide insight into the value of SPR and SPR* forecasts and the information which they transmit as they range through the [0,1] interval. Resolution is high if, on average, different forecasts tend to be followed by different realizations, so that movements in forecast probabilities convey meaningful information. First, compare the NAIVE and CONST forecasts, which, by definition, have zero resolution.¹⁷ 3CD and 3CDa fare somewhat better, but the restrictive nature of the forecasts generated by these

17. For a constant forecast, the grand realization mean is equal to the mean realization in the one cell in which all forecasts lie.

TABLE 6 RES as a Function of Horizon, Various Forecasting Methods

Method	Forecast Horizon					
	1	3	5	7	9	13
Prediction of peaks:						
NAIVE	.00	.00	.00	.00	.00	.00
CONSTANT	.00	.00	.00	.00	.00	.00
	(.02)	(.06)	(.10)	(.14)	(.18)	(.26)
SPR	.00	.02	.04	.06	.07	.08
	(.02)	(.06)	(.10)	(.14)	(.18)	(.26)
SPR*	.00	.01	.03	.06	.07	.07
	(10^{-7})	(.0001)	(.001)	(.005)	(.01)	(.03)
3CD	.00	.00	.01	.02	.02	.02
3CD _a	.00	.02	.04	.06	.06	.04
Prediction of troughs:						
NAIVE	.00	.00	.00	.00	.00	.00
CONSTANT	.00	.00	.00	.00	.00	.00
	(.09)	(.27)	(.45)	(.63)	(.78)	(.93)
SPR	.04	.12	.17	.17	.05	.03
	(.09)	(.27)	(.45)	(.63)	(.78)	(.93)
SPR*	.06	.14	.20	.16	.05	.02
	(.005)	(.07)	(.23)	(.39)	(.95)	(.97)

NOTE.—The CONSTANT probability forecast is a constant (given in parentheses) that minimizes the QPS. The SPR probabilities use this constant probability as a prior (Γ^u or Γ^d , given in parentheses). The SPR* probabilities are generated with the prior transaction probabilities (given in parentheses) that minimize the QPS. The forecast horizon is given in months, and the scoring sample is from December 1948 to December 1986.

methods (e.g., probabilities of only 0.0 or 1.0 for 3CD) results in low RES. The RES is highest for the SPR and SPR* forecasts, reflecting the fact that different forecasts *do* tend to be followed by different realizations, so that movements in the generated probabilities through the [0,1] interval contain useful information. In addition, RES is highest in contractions. Were it not for this fact, SPR and SPR* trough prediction performance in terms of QPS and LPS would be substantially worsened.

VI. Concluding Remarks

We have examined the performance of a Bayesian sequential probability forecasting recursion, with the Composite Index of Leading Indicators. Performance was evaluated in a number of dimensions, including accuracy, calibration, and resolution. One clear result for good forecast performance, as well as proper forecast evaluation, was the need for prior transition probabilities, densities, horizons, and scorings that separated expansions and contractions. Furthermore, this suggests that leading economic indicators might usefully be specialized during expansions for the prediction of peaks and during contractions for the prediction of troughs. In other words, the use of two indexes, an “expansion index” and a “contraction index,” constructed with dif-

ferent components and component weights, could enhance predictive performance.

The sequential probability recursion was the best method of those considered for forecasting turning points, especially given its firm grounding in probability theory and its ability to forecast both peaks and troughs. The *absolute* performance of the sequential probability recursion in terms of accuracy, like all the other forecasting methods, was worse in contractions. Its performance *relative* to other methods, however, was best in contractions. We also examined forecast calibration and resolution, the underlying determinants of accuracy. The calibration analysis showed that most bias could be traced to those probability forecasts near zero or one, illustrating the unavoidable possibilities of “false alarms” and “missed calls.” The sequential probability recursion performed best in terms of resolution, which indicates that useful information is conveyed by movements in its probability forecasts.

Whether the increased resolution afforded by use of the sequential probability recursion in forecasting with the CLI is sufficient to make it the forecasting method of choice depends on the loss function used for accuracy evaluation. Recall, for example, that the QPS may be decomposed into the QPS of a particular constant probability forecast, plus LSB, less RES. More generally, however, one can imagine less restrictive loss functions such as

$$L = f[f(\bar{R}), \text{LSB}, \text{RES}].$$

Even if a linear form is adopted, for example, we need not impose the weights of 1, 1 and -1 which correspond to QPS. To the extent that the partial derivative of L with respect to RES is negative and sufficiently large (in absolute value), the sequential probability recursion can be expected to perform well. Moreover, loss functions that place relatively high (in absolute value) weight on RES may be a good approximation to those of many forecasters and policymakers. To see this, look at figure 1 and ask yourself, “Which would be more useful to *me*, the SPR forecasts shown in the figure, or, for example, a constant-probability forecast (that would appear in the figure as horizontal lines in expansions and contractions)?” Many, for better or for worse, would probably choose the former.

Appendix A

Derivation of the Sequential Probability Recursion

In this appendix, we provide a proof of the sequential probability recursion along the lines of Neftci (1980). Let Z be an integer-valued random variable denoting the value of the time index in the first period after the turning point in

the leading series X . (That is, if $Z = 10$, then the turning point has occurred between periods 9 and 10.) At time t we calculate a probability for the event $Z \leq t$, that is, that by time t the turning point has occurred. We have an a priori probability (at time t) denoted $P(Z \leq t)$. We also receive sequential observations on X , and at time t , we have $t + 1$ observations, denoted $(x_0, x_1, \dots, x_t) \equiv \bar{x}_t$.

The posterior probability of $Z \leq t$ at time t is given immediately by Bayes's rule, as in the text. This can be rewritten as

$$P(Z \leq t | \bar{x}_t) = \frac{\sum_{i=0}^t P(\bar{x}_t | Z = i) P(Z = i)}{\left[\sum_{i=0}^t P(\bar{x}_t | Z = i) P(Z = i) \right] + P(\bar{x}_t | Z > t) P(Z > t)}. \quad (\text{A1})$$

Consider first the numerator, which we denote by A_t :

$$A_t = P(x_0, \dots, x_t | Z = 0) P(Z = 0) + \sum_{i=1}^t P(x_0, \dots, x_t | Z = i) P(Z = i). \quad (\text{A2})$$

Recalling that, if $Z = i$, then (x_0, \dots, x_{i-1}) and (x_i, \dots, x_t) have different (and independent) distributions, we rewrite this as

$$A_t = P(x_0, \dots, x_t | Z = 0) P(Z = 0) + \sum_{i=1}^t [P(x_0, \dots, x_{i-1} | Z = i) P(x_i, \dots, x_t | Z = i) P(Z = i)]. \quad (\text{A3})$$

In period $(t + 1)$, we have (A4):

$$A_{t+1} = P(x_0, \dots, x_{t+1} | Z = 0) P(Z = 0) + \sum_{i=1}^t [P(x_0, \dots, x_{i-1} | Z = i) P(x_i, \dots, x_{t+1} | Z = i) P(Z = i)] + P(x_0, \dots, x_{t+1} | Z = t + 1) P(Z = t + 1). \quad (\text{A4})$$

Making use of the usual factorization, we obtain

$$A_{t+1} = P(x_0, \dots, x_t | Z = 0) P(x_{t+1} | Z = 0, x_0, \dots, x_t) P(Z = 0) + \sum_{i=1}^t [P(x_0, \dots, x_{i-1} | Z = i) P(x_i, \dots, x_t | Z = i) \times P(Z = i) P(x_{t+1} | Z = i, x_i, \dots, x_t)] + [P(x_0, \dots, x_t | Z = t + 1) P(x_{t+1} | Z = t + 1, x_0, \dots, x_t) \times P(Z = t + 1)]. \quad (\text{A5})$$

Thus,

$$\begin{aligned} A_{t+1} &= A_t P(x_{t+1} | Z = t + 1, x_0, \dots, x_t) \\ &\quad + P(Z = t + 1) P(x_0, \dots, x_t | Z = t + 1) \\ &\quad \times P(x_{t+1} | Z = t + 1, x_0, \dots, x_t). \end{aligned} \quad (\text{A6})$$

Now if we write

$$\Pi_t = \frac{A_t}{A_t + B_t}, \quad (\text{A7})$$

so that

$$B_t = P(x_0, \dots, x_t | Z > t) P(Z > t), \quad (\text{A8})$$

then we immediately obtain the recursion

$$B_{t+1} = B_t P(x_{t+1} | x_0, \dots, x_t, Z > t + 1) \frac{P(Z > t + 1)}{P(Z > t)}. \quad (\text{A9})$$

Now note that

$$\Pi_{t+1} = \frac{B_{t+1}}{A_{t+1} + B_{t+1}}, \quad (\text{A10})$$

and substitute (A6) and (A9) to get

$$\Pi_{t+1} = \frac{C}{C + D}, \quad (\text{A11})$$

where $C = [\Pi_t + P(Z = t + 1 | Z > t) (1 - \Pi_t)] [P(x_{t+1} | x_0, \dots, x_t, Z = t + 1)]$, and $D = (1 - \Pi_t) P(x_{t+1} | x_0, \dots, x_t, Z > t + 1) [1 - P(Z = t + 1 | Z > t)]$, after applying (A7) and (A8) and some tedious algebra.

To work the proof for continuous X , we realize that we have implicitly been working with the sample space partition

$$\Delta_t = \{\omega \in \Omega: x_0 < X_0 < x_0 + h, \dots, x_t < X_t < x_t + h\}.$$

Letting $h \rightarrow 0$, we again obtain (A11), except that now the conditional *probabilities*

$$P(x_{t+1} | x_0, \dots, x_t, Z = t + 1)$$

and

$$P(x_{t+1} | x_0, \dots, x_t, Z > t + 1)$$

are replaced by the equivalent conditional *densities*

$$f(x_{t+1} | x_0, \dots, x_t, Z \leq t + 1)$$

and

$$f(x_{t+1} | x_0, \dots, x_t, Z > t + 1).$$

These are, respectively, the densities

$$f^d(x_{t+1}|\bar{x})$$

and

$$f^u(x_{t+1}|\bar{x})$$

given in the text, shifted forward one period.

Appendix B

Business Cycle Chronologies

The NBER dating, year and month, of the postwar business cycles is given in the right-hand column below. The chronology of turning points in the composite index of leading indicators (taken from the *Business Conditions Digest*, chart 1A) is given in the left-hand column. Peaks and troughs are considered to be members of the "old" regime; thus, a peak (trough) is the last observation in an expansion (recession).

TABLE B1

	CLI Turning Points	NBER Turning Points
Trough	1949: 6	1949:10
Peak	1953: 3	1953: 7
Trough	1953:11	1954: 5
Peak	1955: 9	1957: 8
Trough	1958: 2	1958: 4
Peak	1959: 5	1960: 4
Trough	1960:12	1961: 2
Peak	1969: 4	1969:12
Trough	1970:10	1970:11
Peak	1973: 3	1973:11
Trough	1975: 2	1975: 3
Peak	1979: 3	1980: 1
Trough	1980: 5	1980: 7
Peak	1981: 4	1981: 7
Trough	1982: 3	1982:11

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